Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 29, 2004*
Accepted: *May 27, 2004*

Ghost condensation and a consistent infrared modification of gravity

Nima Arkani-Hamed,[a] Hsin-Chia Cheng,[a] Markus A. Luty[abc] and Shinji Mukohyama[a]

[a] *Jefferson Laboratory of Physics, Harvard University*
Cambridge, Massachusetts 02138, U.S.A.
[b] *Physics Department, Boston University*
Boston, Massachusetts 02215, U.S.A.
[c] *Physics Department, University of Maryland*
College Park, Maryland 20742, U.S.A.
E-mail: arkani@carnot.harvard.edu, hcheng@fas.harvard.edu, mluty@physics.umd.edu, mukoyama@schwinger.harvard.edu

Abstract: We propose a theoretically consistent modification of gravity in the infrared, which is compatible with all current experimental observations. This is an analog of the Higgs mechanism in general relativity, and can be thought of as arising from ghost condensation — a background where a scalar field ϕ has a constant velocity, $\langle\dot{\phi}\rangle = M^2$. The ghost condensate is a new kind of fluid that can fill the universe, which has the same equation of state, $\rho = -p$, as a cosmological constant, and can hence drive de Sitter expansion of the universe. However, unlike a cosmological constant, it is a physical fluid with a physical scalar excitation, which can be described by a systematic effective field theory at low energies. The excitation has an unusual low-energy dispersion relation $\omega^2 \sim \vec{k}^4/M^2$. If coupled to matter directly, it gives rise to small Lorentz-violating effects and a new long-range $1/r^2$ spin dependent force. In the ghost condensate, the energy that gravitates is not the same as the particle physics energy, leading to the possibility of both sources that can gravitate and *anti*-gravitate. The newtonian potential is modified with an oscillatory behavior starting at the distance scale $M_{\rm Pl}/M^2$ and the time scale $M_{\rm Pl}^2/M^3$. This theory opens up a number of new avenues for attacking cosmological problems, including inflation, dark matter and dark energy.

Keywords: Classical Theories of Gravity, Cosmology of Theories beyond the SM.

Contents

JHEP05(2004)074

1. Introduction and summary

Gravity remains the most enigmatic of all the fundamental interactions in nature. At the Planck scale, large quantum fluctuations signal a breakdown of the effective field theory description of gravity in terms of general relativity, leading to the expectation that gravity and our notions of space and time must be radically altered at short distances. Meanwhile, continuing experimental probes of gravity at *large* distances reveal strange phenomena at many length scales, from the flattening of galactic rotation curves to the accelerating universe. Traditionally, these phenomena are explained by invoking new sources of matter and energy, such as dark matter and a tiny cosmological constant. It is however worth contemplating the possibility that gravity itself is changing in the infrared in some way that might address these mysteries. This approach is further motivated by the cosmological constant problem, which seems to be associated with extreme infrared physics.

As a first step in this program, it is an interesting theoretical challenge to find a theoretically consistent and experimentally viable modification of gravity in the infrared (IR). Such attempts have been made in the past [1]–[11]. Many of these theories are in one way or another varieties of scalar-tensor theories of gravity, and therefore are not true modifications of gravity. For example, the the long-range interactions between two masses in these theories is at most changed by a constant factor. Much more interesting ideas are those that modify gravity interactions in a dramatic way at large distances, such as the Fierz-Pauli theory of massive gravity [1], and the Dvali-Gabadadze-Porrati (DGP) brane-world models with infinite volume extra dimensions [5]. However, it is not yet entirely clear that these theories are experimentally viable. In both of these theories, there is an additional scalar degree of freedom, π, interacting with matter with gravitational strength. In the case of massive gravity it is the helicity zero longitudinal mode of the graviton [12, 13, 14], and in the DGP model it is the "brane-bending" mode [15] (see also [16]). Because of the presence of the π's, the theory reduces to scalar-tensor theory of gravity, rather than general relativity, at distances smaller than the length scale of IR modification. Furthermore, the π's become strongly coupled in the ultraviolet (UV) at a scale $\Lambda_{\rm UV}$ determined in terms of the IR modification scale $\Lambda_{\rm IR}$ and the Planck scale $M_{\rm Pl}$. At $\Lambda_{\rm UV}$, the effective theory breaks down and the physics is sensitive to the unknown UV completion of the theory. That such a UV scale should exist is not surprising. After all, even in non-abelian gauge theories, if we modify the theory in the IR by giving the gauge boson a mass m, this introduces a new degree of freedom, the longitudinal component of the gauge field. This degree of freedom becomes strongly coupled in the UV at scale of order $4\pi m/g$, determined by the scale of IR modification and the gauge coupling g. However, the situation is both qualitatively and quantitatively different in the gravitational case. The highest strong coupling scale $\Lambda_{\rm UV}$ can be pushed in all of the above cases is [14, 15],

$$\Lambda_{\rm UV} \sim (\Lambda_{\rm IR}^2 M_{\rm Pl})^{1/3}\,. \tag{1.1}$$

For $\Lambda_{\rm IR} \sim H_0$, today's Hubble scale, we have $\Lambda_{\rm UV}^{-1} \sim 1000$ km, which is much larger than the distances to which we have probed gravity. Roughly speaking, this happens because gravity is being coupled to a sector that is sick in the limit as gravity is turned off ($M_{\rm Pl} \to \infty$). The π's have no kinetic term in this limit, even though they have cubic and higher order self-interactions. The theory only becomes tenuously healthy due to the coupling with gravity: mixing with gravity generates small kinetic terms for π of the correct sign (in flat space), but leads to strong coupling physics at low energies. This does not necessarily mean that massive gravity or the DGP model cannot describe the real world, only that it appears necessary to make nontrivial assumptions about the UV completion of the theory.

In this paper, we present a fully controllable and calculable theory where gravity is modified in the infrared, in a way that avoids these strong coupling issues. It is useful, though not necessary, to view this as arising from "ghost condensation." A ghost condensate is a new kind of fluid that can fill the universe. It has the equation of state $p = -\rho$, just like a cosmological constant Λ, and can therefore gives rise to a de Sitter phase of the universe even if $\Lambda = 0$. But the ghost condensate is *not* a cosmological constant, it is a physical fluid with a physical scalar excitation. This background can be thought of as arising from a theory where a real scalar field ϕ is changing with a constant velocity

$$\langle \dot\phi \rangle = M^2 \qquad \text{or} \qquad \langle \phi \rangle = M^2 t\,, \tag{1.2}$$

and there is a physical scalar excitation π around this background,

$$\phi = M^2 t + \pi \,. \tag{1.3}$$

We assume that the field ϕ has a shift symmetry

$$\phi \mapsto \phi + a \,, \tag{1.4}$$

so it is derivatively coupled, and the quadratic lagrangian for ϕ begins with a wrong-sign kinetic term [in $(+ - --)$ signature]

$$\mathcal{L}_\phi = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \cdots \,. \tag{1.5}$$

The wrong-sign kinetic term means that the usual background with $\langle\phi\rangle = 0$ is unstable. However, as is familiar from tachyon condensation, we will see that higher order terms can stabilize a background where $\dot{\phi} \neq 0$. Note that this background breaks Lorentz invariance spontaneously — there is a preferred frame where ϕ is spatially isotropic. This is not fundamentally different from the way in which the cosmic microwave background radiation (CMBR), or any other cosmological fluid, breaks Lorentz invariance. What is novel about the ghost condensate is that, unlike other cosmological fluids (but like a cosmological constant), it does not dilute as the universe expands.[1]

We will construct a systematic derivative expansion governing the low-energy physics of the excitation π about the background. As we will see, these excitations have two important properties, that combine to give rise to a number of surprising new phenomena in the IR.

First, the low-energy effective action for π has the universal form

$$S \sim \int d^4x \left[\frac{1}{2}\dot{\pi}^2 - \frac{1}{2M^2}(\nabla^2\pi)^2 + \cdots\right], \tag{1.6}$$

so that the π's have a low-energy dispersion relation of the form

$$\omega^2 \sim \frac{k^4}{M^2} \,. \tag{1.7}$$

(Throughout this paper k represents the spatial 3-momentum and ∇ represents the spatial gradient. In this introductory section, we will not be careful about $\mathcal{O}(1)$ coefficients in front of various terms.) Ordinarily the dispersion relation for light excitations is of the form $\omega^2 \propto k^2$. Here, there is no k^2 term at all, so the excitations do not move with any fixed speed. Instead, the group velocity is $v^2 \sim k^2/M^2$, so that larger lumps of π move more slowly than smaller lumps. Note that this is not a Lorentz-invariant dispersion relation, as is to be expected since the background breaks Lorentz invariance.

[1] Attempts to modify gravity with a dynamical preferred frame violating Lorentz invariance have been made, e.g., most recently in ref. [6], but with different physical conclusions.

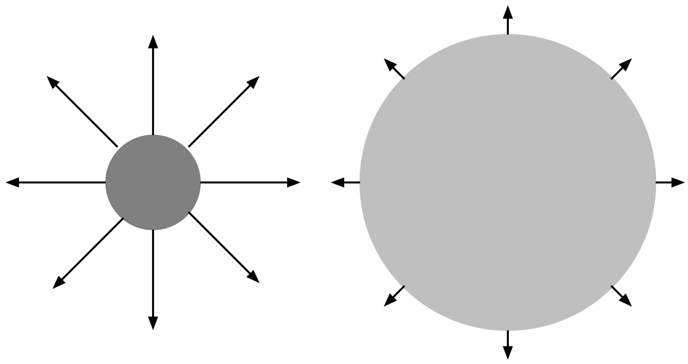

Figure 1: Excitations π about the ghost condensate have a dispersion relation $\omega^2 \propto k^4$, so that small lumps of π expand more quickly than larger lumps of π.

Second, there are two distinct kinds of energy in these backgrounds: a "particle physics" energy and a "gravitational" energy, and they are not the same. This happens because the background $\langle\phi\rangle = M^2 t$ breaks time translational invariance. It also breaks the ϕ shift symmetry, but the diagonal combination is left unbroken to generate the "time" translations in this background. The Noether charge associated with this unbroken symmetry is what an experimentalist coupled to π would call the conserved energy in this background, and this "particle physics" energy density takes the form

$$\mathcal{E}_{\rm pp} \sim \frac{1}{2}\dot{\pi}^2 + \frac{(\nabla^2\pi)^2}{2M^2} + \cdots , \tag{1.8}$$

and is manifestly positive for small fluctuations. On the other hand, the energy density that couples to gravity is T_{00}, which is associated with the broken time-translation symmetry, and is *not* the same as $\mathcal{E}_{\rm pp}$. Indeed,

$$\mathcal{E}_{\rm grav} = T_{00} \sim M^2\dot{\pi} + \cdots \tag{1.9}$$

begins at *linear* order in $\dot{\pi}$, therefore lumps of π can either gravitate or *anti*-gravitate depending on the sign of $\dot{\pi}$! In fact, the "particle physics" energy is nothing other than the inertial mass, while the "gravitational energy" is the gravitational mass, so we can say that the π excitations maximally violate the equivalence principle. It is worth mentioning that excitations of π with exotic gravitational properties are not easily produced if π couples only to gravity. In fact, we find that turning on π gravitationally requires long distances and time scales, as discussed below.

The standard model fields do not need to couple directly to the ghost sector. However, if there *is* a direct coupling, then the leading interaction (a derivative coupling of ϕ to the axial vector currents) gives rise to potentially observable consequences, including a splitting between particle and antiparticle dispersion relations, and a new spin-dependent inverse-*square* law forces mediated by π exchange. This inverse square law behavior is a direct consequence of the $\omega^2 \propto k^4$ dispersion relation.

Even if the standard model fields have no direct couplings to the ghost sector, they will indirectly interact with it through gravity, and the propagation of gravity through

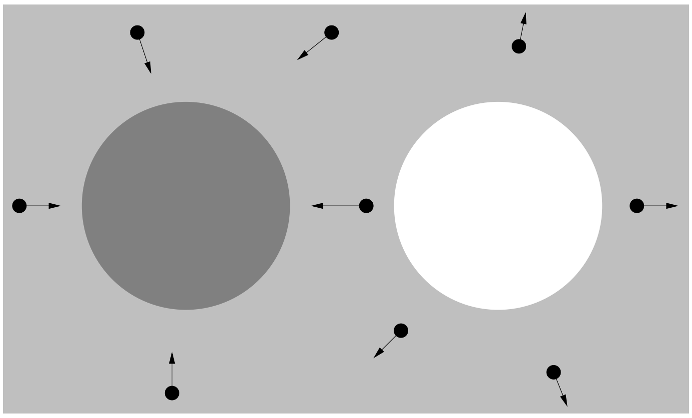

Figure 2: Lumps of π can either gravitationally attract test particles (in regions where $\dot{\pi} > 0$) or repel them (where $\dot{\pi} < 0$).

the ghost condensate gives rise to a fascinating modification of gravity in the IR. As an example, consider a cloud of dust that quickly collapses to form a star or a planet at some time $t = 0$. At linear order, the gravitational potential around this planet is modified at a length scale r_c, but only after waiting a much longer time t_c, where

$$r_c \sim \frac{M_{\rm Pl}}{M^2}\,, \qquad t_c \sim \frac{M_{\rm Pl}^2}{M^3}\,. \tag{1.10}$$

Starting at time t_c the potential becomes *oscillatory* at distances of order r_c, and the modification reaches out to much larger distances $r \gg r_c$ only after waiting an even longer time $t \sim (r/r_c)t_c$. Furthermore, in a flat background, the ghost condensate has a Jeans-like instability at long wavelengths, so that the modulation of the potential begins growing exponentially at late time $t \gg t_c$. As with ordinary fluids, however, this Jeans instability disappears due to Hubble friction in a cosmological background (such as de Sitter space) with $H > t_c^{-1}$. In this case, at late time $t \gg t_c$, the potential looks like an oscillatory modulation of $1/r$ such as shown in figure 3.

There are a number of interesting ranges of parameters for the mass scale M associated with the ghost condensation. If the ghost condensate is to give the observed acceleration of the universe today, even with $\Lambda = 0$, then naturalness suggests

$$M \sim 10^{-3}\,\text{eV}\,. \tag{1.11}$$

In this case, the distance scale where gravity is modified is $r_c \sim H_0^{-1}$, but the time it takes to see this modification is $t_c \gg H_0^{-1}$! So no modifications of gravity can be seen directly, and no *cosmological* experiment can distinguish the ghost-driven acceleration from a cosmological constant. If the standard model has direct coupling to the ghost sector, we can still hope to detect tiny Lorentz-violating effects and long-range spin-dependent inverse square force laws. It is also possible that M is much larger; this could be the remnant of ghost condensation triggering an earlier de Sitter epoch in the universe, to drive inflation [17]. For instance, we can have the parameters

$$M \sim 10\,\text{MeV}\,, \qquad t_c \sim H_0^{-1}\,, \qquad r_c \sim 1000\,\text{km}\,, \tag{1.12}$$

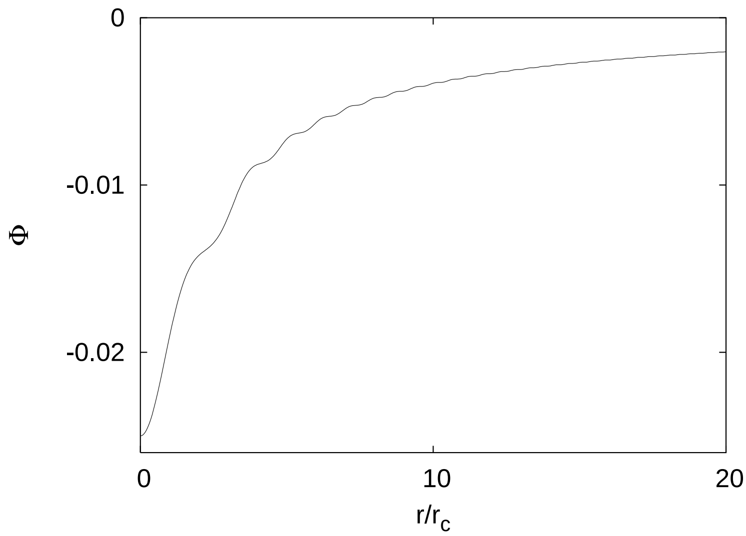

Figure 3: Example of a late-time newtonian potential.

so that we begin to start seeing an oscillatory modulation of newtonian gravity at a distance of ~ 1000 km, but only recently in the history of the universe!

This physics is an exact analog of the Higgs mechanism for gravity. Recall that general relativity is a gauging of spacetime translations. In particular the time diffeomorphism symmetry is spontaneously broken by our background $\langle\phi\rangle = M^2 t$, and therefore we should expect gravity to become "massive." There are two differences with the familiar gauge theory case. First, our background breaks Lorentz invariance, and therefore we do not get the massive graviton of a Lorentz invariant theory, which would have 5 polarizations. In fact we are only adding one additional physical degree of freedom, which can be thought of as the scalar excitations π. Second, while in the gauge theory case we get a Yukawa-like exponential suppression of the potential at large distances, in our case we get an oscillatory modulation instead — basically because this is a *negative* mass-squared term for the graviton. This difference in sign is to be expected in all comparisons of gravity and gauge theory, physically arising for the same reason that like charges repel in gauge theory and attract in gravity. It is also analogous to the fact that fluctuations of a charged fluid acquire a positive plasma frequency $\omega_p^2 > 0$, while gravity induces an $\omega_J^2 < 0$ signalling the Jeans instability.

We have described the physics so far in the language of gravity propagating through a new fluid. This is like the description of massive gauge theories in terms of propagation through the Higgs condensate. As familiar from the gauge theory, there is also a "unitary gauge" description of the physics, where the excitation π is set to zero ("eaten"), and where the interpretation of the theory directly as an IR modification of gravity is clear. The unitary gauge is arrived at by choosing the time coordinate t such that

$$\phi(t, x) = M^2 t\,. \tag{1.13}$$

That is, ϕ is taken to be the clock. This is equivalent to making a diffeomorphism to set $\pi = 0$. In this gauge, there are terms in the action that are not fully diffeomorphism invariant. At quadratic level for the fluctuations $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ around flat space these new terms in the lagrangian take the form

$$\Delta\mathcal{L} = \alpha_0 M^4 h_{00}^2 - \alpha_1 M^2 K_{ij}K_{ij} - \alpha_2 M^2 (K_{ii})^2 \,, \tag{1.14}$$

where

$$K_{ij} = \frac{1}{2}(\partial_0 h_{ij} - \partial_i h_{0j} - \partial_j h_{0i})\,, \tag{1.15}$$

and $\alpha_{0,1,2}$ are $\mathcal{O}(1)$ coefficients. The fully non-linear action will be presented in section 6. These terms are clearly "mass" terms and unusual kinetic terms for $h_{\mu\nu}$.

It is also instructive to understand the counting of degrees of freedom in this theory in the $(3+1)$-split ADM language [18], where the metric is split into $N \leftrightarrow g^{00}$, $N_i \leftrightarrow g_{0i}$, and $\gamma_{ij} \leftrightarrow g_{ij}$. Usually in GR, both N and N_i are not dynamical fields, they are Lagrange multipliers, and their equations of motion give $1+3=4$ constraints on the γ_{ij}, reducing the 6 degrees of freedom in γ_{ij} to the two physical polarizations of the graviton. Our background breaks time-diffeomorphisms, but leaves (time-dependent) spatial diffeomorphisms intact. As a consequence, the N_i are still Lagrange multipliers enforcing 3 constraints. However, N is no longer a Lagrange multiplier, and the N equation of motion then simply determines N. There are therefore $6-3=3$ physical degrees of freedom — we have added one degree of freedom. Like all gauge "symmetries," diffeomorphisms are not a symmetry, they are a useful redundancy of description. The physical issue is what the degrees of freedom and their dynamics are. The additional degree of freedom is not manifest in the unitary gauge description of the physics, and it is most convenient to explicitly introduce the Goldstone field π that non-linearly realizes the symmetry. In this way, arriving at this theory through the background $\langle\phi\rangle = M^2 t$ of a scalar is the analog of a linear sigma model description of gauge symmetry breaking. This is not necessary however — we could have simply started with the theory of the π and its (non-linear) transformations under time diffeomorphisms explicitly, just as we can directly write the chiral lagrangian for pions without any reference to an underlying linear sigma model. This line of thought suggests another way in which our model might arise from a more fundamental underlying theory. One of the well-known conceptual difficulties in putting quantum mechanics and gravity is the problem of time: quantum mechanics picks out a direction of time, in conflict with general covariance. Obviously this problem must be resolved at some zeroth order, however, if any remnant of this clash remains at low energies, it will be described by our theory at long distances and times.

In the rest of the paper we will discuss this physics in detail. In section two we study ghost condensation in general, first in the absence of gravity, and then in a cosmological Friedmann-Robertson-Walker (FRW) background. As we will see Hubble friction in an expanding universe is crucial in determining the background. We discuss how de Sitter phases of the universe can arise from ghost condensation in section 3. In section 4, we discuss the low-energy effective action for the fluctuations around the ghost condensate in the absence of gravity, and perform the correct power-counting analysis to show that the theory is (non-trivially) a sensible, quantum-mechanically stable effective theory at low energies, so the π's are a healthy sector even in the absence of gravity. In section 5, we discuss the signals that would arise from a direct coupling of the standard model fields to

JHEP05(2004)074

the ghost sector. In section 6, we systematically construct the low-energy effective theory for the ghost condensate coupled to gravity. In section 7, we study the modification of gravity in the ghost condensate in flat space at linear order, arising from mixing between the scalar sector of gravity and the π excitations. Here we uncover the Jeans instability of the fluid at large wavelengths, and the oscillatory modulation of the newtonian potential at large distances and late times. We also make some brief comments on the interesting question of corrections to non-linear gravity. In section 8, we repeat this analysis for a de Sitter background, where the Jeans instability is removed. We end with a discussion of some of the many open questions and avenues for further research.

2. Ghost condensation

2.1 Generalities

Consider a scalar field ϕ with a shift symmetry $\phi \mapsto \phi + a$ ensuring that it always appears derivatively in the action. Assume that it has a wrong-sign quadratic kinetic term, i.e., it is a *ghost* field:

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \cdots . \tag{2.1}$$

Actually, the overall sign of the lagrangian is completely unphysical, so what is actually relevant is that once a choice has been made for the sign of the lagrangian for, say, the standard model and gravity sector, a ghost is a field with opposite sign kinetic term. The difficulty is then that, with both kinds of fields, the energy of the theory is unbounded from below, and therefore the vacuum $\langle\phi\rangle = 0$ is unstable. Vacuum decay will inevitably happen quantum-mechanically. Consider the minimal case where ϕ is only coupled gravitationally. Kinematically, nothing prevents the vacuum from decaying to a pair of ϕ's and a pair of gravitons. There is more and more phase space for this for higher energy gravitons (and more negative energy ϕ's), but if the ϕ theory has a UV cutoff at a scale M, then the rate for vacuum decay per unit volume is $\Gamma \sim \Lambda_{\rm UV}^8/M_{\rm Pl}^4$. For $\Lambda_{\rm UV} \gtrsim 10^{-3}\,{\rm eV}$, this decay will happen within our Hubble volume within a Hubble time. Other possible decays and constraints were considered in ref. [19, 20].

Let us compare this with the more familiar example of a tachyon, which is a scalar with negative mass squared. In this case the vacuum $\langle\phi\rangle = 0$ is also unstable. Even if classically we can sit at the point $\phi = 0$ forever, quantum-mechanically we inevitably roll off the top of the potential with a time scale of order m^{-1}. In the tachyon case, we are accustomed to the idea of tachyon condensation: if there are higher order terms that turn the potential around, e.g., $V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \cdots$ with $\lambda > 0$, then there can be a stable vacuum where $\langle\phi\rangle \neq 0$.

We can ask whether it is possible to have find a stable "ghost condensate" vacuum for a theory with wrong-sign kinetic terms. In fact, general relativity can be viewed as such a theory. The *Vierbein* and spin connection form the gauge fields for $ISO(3,1)$, the group of Lorentz transformations and translations on 4-dimensional Minkowski space. The gauge group is noncompact, so fluctuations about the vacuum $e^a_\mu = 0$ include ghosts, but fluctuations about nontrivial vacua such as flat space $\langle e^a_\mu\rangle = \delta^a_\mu$ are stable. In our

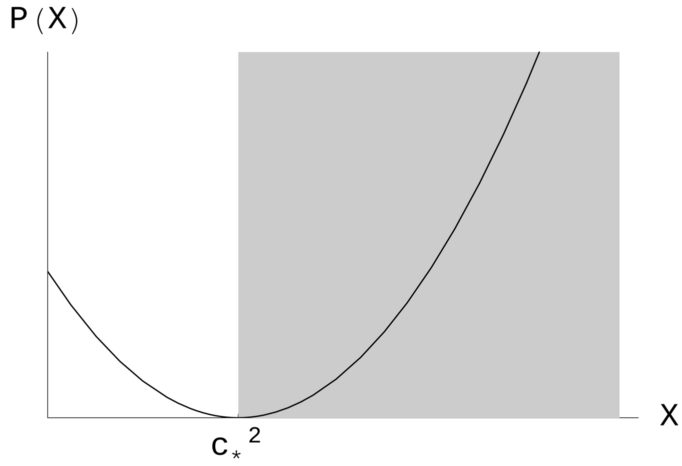

Figure 4: The function P. The shaded region is the one for which the small fluctuations about the background have healthy two-derivative time and spatial kinetic terms.

theory with a single scalar, we can imagine that there are other terms such as $(\partial\phi)^4$ in the lagrangian that "stabilize" the ghost in the same way the ϕ^4 term stabilizes the tachyon. One immediate difference with the tachyon case is that all of these higher order terms are non-renormalizable; if these terms are to become as important as the $(\partial\phi)^2$ term as they must in order to stabilize the theory, then it appears that a description in terms of effective field theory is not possible. As we will see this is in fact not the case, and there is a systematic low-energy expansion for fluctuations around the ghost condensate. But let us press on for a bit, and start by assuming that the lagrangian has the form

$$\mathcal{L} = M^4 P(X)\,, \tag{2.2}$$

where $X = \partial^\mu\phi\partial_\mu\phi$. We have now chosen ϕ to have the dimension of length, so that X is dimensionless, while M is a constant with the dimension of mass. Note that we are neglecting terms involving more than one derivative acting on ϕ; we will come back to these terms below. A typical form for the function P we are considering is shown in figure 4.

The equations of motion that follow from this lagrangian are

$$\partial_\mu\left[P'(X)\partial^\mu\phi\right] = 0\,. \tag{2.3}$$

From this we see that we obtain a solution provided that $\partial_\mu\phi$ = constant. If $\partial_\mu\phi$ is time-like, there is a Lorentz frame where

$$\phi = c\,t\,, \tag{2.4}$$

where c is a dimensionless constant. These solutions are actually the maximally symmetric solutions, since they break the ϕ shift symmetry and the time translation symmetry down to an unbroken diagonal shift symmetry.

Let us now consider small fluctuations about these solutions

$$\phi = c\,t + \pi\,. \tag{2.5}$$

The lagrangian for quadratic fluctuations is

$$\mathcal{L} = M^4 \left\{ \left[P'(c^2) + 2c^2 P''(c^2) \right] \dot{\pi}^2 - P'(c^2)(\nabla\pi)^2 \right\}. \tag{2.6}$$

We see that the fluctuations have the time and spatial kinetic terms of the usual signs provided that c is such that

$$P'(c^2) > 0\,, \qquad P'(c^2) + 2c^2 P''(c^2) > 0\,. \tag{2.7}$$

Around these backgrounds the excitations of π have positive energy, and so any weak coupling of this sector to other sectors (such as gravity and the standard model) will not suffer from the violent UV instability associated with the ghosts around the usual background with $\partial_\mu\phi = 0$. This does not uniquely fix c, any value of c satisfying these conditions is stable against small fluctuations (see figure 4).

More generally, we have additional terms in the effective lagrangian arising from terms with two or more derivatives acting on ϕ:

$$\mathcal{L} = M^4 P(X) + M^2 S_1(X)(\Box\phi)^2 + M^2 S_2(X)\partial^\mu\partial^\nu\phi\partial_\mu\partial_\nu\phi + \cdots\,. \tag{2.8}$$

(We have assumed for simplicity here that there is a $\phi \mapsto -\phi$ symmetry that forbids terms with an odd number of ϕ's in the lagrangian). With these additional terms included, there is still a solution for any value of c, since the ϕ equation of motion always takes the form

$$\partial_\mu \Big[\text{function of } \partial\phi,\, \partial^2\phi,\, \ldots \Big] = 0\,. \tag{2.9}$$

The quadratic lagrangian for π now includes terms with more derivatives on the π's. We will see below that Hubble friction drives us to a background where the spatial kinetic terms coming from P' vanish and that the leading spatial kinetic terms are fourth order and arise from the $S_{1,2}$ terms in eq. (2.8).

We can now understand why it is possible to have a good low-energy effective field theory for the physics, despite the appearance of the non-renormalizable interactions $P(\partial\phi)^2, S((\partial\phi)^2)$ and so on. The point is that the background sits at some specific value of c, and therefore does not sample large regions of the functions $P, S_{1,2}, \ldots$. Terms with $n > 1$ derivatives acting on ϕ give terms of order $\partial^n\pi$, which are higher order in the derivative expansion. After Taylor expanding the coefficient functions $P, S_{1,2}, \cdots$, we obtain an effective lagrangian with a finite number of parameters that controls small fluctuations about a given background.

Another way to look at this is to note that the background is characterized by a symmetry breaking pattern. For example, the ghost condensate breaks Lorentz transformations down to spatial rotations. The π field is the Goldstone boson and must non-linearly realize the full Poincare symmetry of the theory, and the π effective lagrangian is highly constrained by symmetry considerations. This point of view will be more fully developed below.

JHEP05(2004)074

2.2 Hubble friction

So far, we have a continuum of possible backgrounds labelled by the parameter c. The situation changes in the presence of gravity and an expanding universe. Gravity is coupled to the effective lagrangian of the previous section via the usual minimal coupling:

$$\mathcal{L} = \sqrt{-g}M^4P(X) + \cdots, \tag{2.10}$$

where $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Again, we begin by ignoring the higher order terms $S_{1,2}$, *etc.* The energy-momentum tensor of the ghost condensate is

$$T_{\mu\nu} = -M^4P(X)g_{\mu\nu} + 2M^4P'(X)\partial_\mu\phi\partial_\nu\phi\,. \tag{2.11}$$

In general, we expect the background to be an expanding FRW universe, with metric

$$ds^2 = dt^2 - a^2(t)d\Omega^2\,, \tag{2.12}$$

where $d\Omega^2$ is the spatial metric for a maximally symmetric 3-dimensional space. For ϕ depending only on t, the ϕ equation of motion in these coordinates is

$$\partial_t\left[a^3P'(\dot\phi^2)\dot\phi\right] = 0\,. \tag{2.13}$$

Note that because this is the ϕ equation of motion, it holds even if some other sector is dominating the expansion of the universe.

From here we see that

$$\dot\phi P(\dot\phi^2) = \frac{\text{const}}{a^3(t)} \to 0 \qquad \text{as} \qquad a \to \infty\,. \tag{2.14}$$

So, in the far future as $a \to \infty$, we are driven to either

$$\dot\phi = 0 \qquad \text{or} \qquad P'(\dot\phi^2) = 0\,, \tag{2.15}$$

depending on the initial condition. The first option is what we are familiar with from massless scalar fields in an expanding universe — the scalar velocity redshifts to zero as the universe expands. However, the point with $\dot\phi = 0$ is the one corresponding to the unstable theory with the wrong sign kinetic term. The other possibility is that the theory is driven to $\dot\phi = c_*$, with $P'(c_*^2) = 0$. If $P''(c_*^2) > 0$, the fluctuations about this point are stable with suitable higher derivative terms as we will see momentarily.

These conclusions are robust under inclusion of the higher-order terms $S_{1,2}, \ldots$. In the case where the universe becomes flat in the asymptotic future, $\dot\phi = c$ implies that $\nabla_\mu\nabla_\nu\phi = 0$, and so all of the higher order terms vanish in the equation of motion and we are still driven to the point where $P' = 0$. If the asymptotic metric is de Sitter space with a Hubble constant H, it is no longer true that for $\dot\phi = c$, all higher covariant derivatives vanish; for instance, $\Box\phi = 3Hc$, so the new terms make non-vanishing contributions to the equation of motion. In this case, we are not driven to $P' = 0$ exactly, but the important point is that Hubble friction singles out *some* value of c_* with $P'(c_*^2) \sim$ powers of H/M. These are small for $H/M \ll 1$, which is required by validity of the low-energy effective theory.

2.3 Low energy dispersion relation

We have seen that gravity forces us into an extremum $P'(c_*^2) = 0$ (for an asymptotically flat spacetime). Recall that the spatial kinetic terms for π from the expansion of P about the background are proportional to P', so if $P' = 0$ there is no $(\nabla\pi)^2$ term in the quadratic lagrangian for π. However, the fluctuations can still be stabilized by $(\nabla^2\pi)^2$ terms, which arise from the higher order terms proportional to $S_{1,2}$,

$$\Delta\mathcal{L} = M^2\left[S_1(c_*^2) + S_2(c_*^2)\right](\nabla^2\pi)^2 + \cdots . \tag{2.16}$$

This stabilizes fluctuations about $\phi = c_* t$ provided that $S_1(c_*^2) + S_2(c_*^2) < 0$. The quadratic lagrangian for π is then

$$\mathcal{L} = \frac{1}{2}M^4\dot{\pi}^2 - \frac{1}{2}\bar{M}^2(\nabla^2\pi)^2 , \tag{2.17}$$

where we have put $P''(c_*^2) = \frac{1}{4}$, which can always be achieved by rescaling between P and M^4, and we have defined $\bar{M}^2 = -2M^2(S_1(c_*)^2 + S_2(c_*^2))$. The π modes then have the low-energy dispersion relation

$$\omega^2 = \frac{\bar{M}^2}{M^4}k^4 . \tag{2.18}$$

This conclusion also holds for an asymptotically de Sitter space. As an example, consider the lagrangian of the form

$$\mathcal{L} = \sqrt{-g}M^4\left[P(X) + Q(X)R(\Box\phi)\right]. \tag{2.19}$$

For ϕ depending only on t, the equation of motion (2.13) is modified to

$$\partial_t\left\{a^3\left[\left(P'(\dot{\phi}^2) + Q'(\dot{\phi}^2)R(\ddot{\phi} + 3H\dot{\phi})\right)2\dot{\phi} - \partial_t(Q(\dot{\phi}^2)R'(\ddot{\phi} + 3H\dot{\phi})\right)\right]\right\} = 0 . \tag{2.20}$$

As $a \to \infty$ we have

$$\left[P'(\dot{\phi}^2) + Q'(\dot{\phi}^2)R(\ddot{\phi} + 3H\dot{\phi})\right]2\dot{\phi} \to 0 \tag{2.21}$$

$$Q(\dot{\phi}^2)R'(\ddot{\phi} + 3H\dot{\phi}) \to \text{const.} \tag{2.22}$$

Expanding in powers of π, we find that the coefficient of the $(\nabla\pi)^2$ term is exactly $-M^4(P'(\dot{\phi}^2) + Q'(\dot{\phi}^2)R(\ddot{\phi} + 3H\dot{\phi}))$, which is driven to zero (for $\dot{\phi} \neq 0$) by the expansion of the universe. The generality of the k^4 dispersion relation is easier to see from the non-linear realization formulation, to be discussed in section 6.

If the universe did not start in the special configuration $\dot{\phi} = c_*$, the $(\nabla\pi)^2$ term will not vanish exactly today and its size depends on the history of the universe. As we will see later, even in the absence of the $(\nabla\pi)^2$ term there will be a k^2 piece induced by mixing with gravity in the dispersion relation. The coefficient of the $(\nabla\pi)^2$ term can be made negligibly small compared with the gravity-induced piece if it was driven to zero e.g. by a period of inflation in the early universe. On the other hand, it would also be interesting to study the effects of a significant $(\nabla\pi)^2$ term that may still remain in the current universe.

JHEP05(2004)074

2.4 Two kinds of energy and (anti)gravity

Our background

$$\langle\phi\rangle = c_* t \tag{2.23}$$

spontaneously breaks time translation invariance. It also breaks the shift symmetry $\phi \to \phi + c$. However, a diagonal sum of these two shift symmetries is left unbroken as the time translation in the background. The combination

$$\mathcal{E}_{\rm pp} = \int d^3x\, T_{00} - c_* Q \tag{2.24}$$

of the Noether charge $\int d^3x\, T_{00}$ associated with time translations, and the charge Q associated with the ϕ shift symmetry, is the conserved energy in this background. The usefulness of $\mathcal{E}_{\rm pp}$ is that it is the conserved energy that an experimentalist would infer from the dynamics of the π fluctuations

$$\mathcal{E}_{\rm pp} = \int d^3x \left[\frac{1}{2}M^4\dot{\pi}^2 + \frac{1}{2}\bar{M}^2(\nabla^2\pi)^2 + \cdots\right]. \tag{2.25}$$

It is the conservation of this quantity that ensures the stability of the vacuum against small fluctuations. On the other hand, this is *not* the energy T_{00} that couples to gravity. Indeed, from the expression for $T_{\mu\nu}$ we can infer that

$$T_{00} = c_*^3 M^4 \dot{\pi} + \cdots\,. \tag{2.26}$$

This begins at *linear* order in $\dot{\pi}$. Therefore, T_{00} can be either positive or negative, depending on the sign of $\dot{\pi}$, and localized excitations of the ghost condensate can give rise to antigravity! Together with the unusual dispersion relation for π, this will be the root of the exotic infrared phenomena we will find below.

3. de Sitter expansion from ghost condensation

Our background spontaneously breaks Lorentz invariance, since there is a preferred frame in which ϕ is spatially isotropic. This is not conceptually any different from the Lorentz violation due to any cosmological fluid that fills the universe, like the cosmic microwave background radiation. Indeed, because of Hubble friction it is natural that this frame coincides with the frame in which the cosmic microwave background is isotropic. There is however an important difference between the ghost condensate and conventional forms of matter and radiation: it does not redshift away as the universe expands. Indeed, in the far future the energy-momentum tensor becomes

$$T_{\mu\nu} \to -g_{\mu\nu} M^4 P(c_*^2)\,, \tag{3.1}$$

i.e., precisely the form of a cosmological constant! If $P(c_*^2) < 0$, the universe will approach a de Sitter metric. However the ghost condensate is *not* a cosmological constant, it is a physical fluid with a physical scalar excitation.

JHEP05(2004)074

The cosmological constant in this theory is simply $\Lambda = -M^4 P(0)/M_{\rm Pl}^2$. Even if this vanishes, the ghost condensate can drive the current acceleration of the universe if $M^4 P(c_*^2) \sim -(10^{-3}\,{\rm eV})^4$, and assuming that there are no very large or small dimensionless parameters in c, this implies that $M \sim 10^{-3}\,{\rm eV}$. It is also possible for the ghost condensate to drive an early inflationary phase of the universe, as will be discussed in ref. [17]. The fact that accelerated expansion of the universe can be driven by the kinetic terms was discussed before, under the names of "k-inflation" [21] and "k-essence" [22]. In those works, $\dot{\phi}$ is required to move over a large range of the kinetic function, which is truncated to low orders or assumed to be of some special form. These models therefore require some knowledge of the fundamental theory and are not based on consistent effective field theories.

If ϕ is not exactly at the solution $\dot{\phi} = c_*$ where $P'(c_*^2) = 0$, then we have

$$P'(\dot{\phi}^2) \sim \frac{1}{a^3}\,. \tag{3.2}$$

From the energy-momentum tensor of the ghost condensate eq. (2.11), we see that a nonzero P' couples to gravity like non-relativistic matter, namely $T_{00} \sim M^4 P'$, $T_{ij} = 0$. Therefore in the approach to its final position at $\dot{\phi} = c_*$, the ghost condensate sources gravity like non-relativistic matter. This raises the prospect that the ghost condensate may contribute to dark matter as well as the vacuum energy, hinting that these phenomena may both have their origin in a modification of gravity at large distance scales. This possibility will be investigated in future work.

4. Low-energy effective theory

We would now like to examine the low-energy effective theory for π in more detail, in the absence of gravity. We will consider the theory expanded around the background $P'(c_*^2) = 0$, which we are driven to by Hubble friction. We show that this theory is truly healthy in the absence of gravity, in contrast to the analogous scalar sectors in DGP and massive gravity, which only become healthy due to the coupling to gravity. In particular, we will address the regime of validity of the effective theory and the question of stability.

Expanding $X = c_*^2 + 2c_*\dot{\pi} + \dot{\pi}^2 - (\nabla\pi)^2$, we can see that the low-energy effective action for the π's begins as

$$S = \int d^3x\,dt \left[\frac{1}{2}M^4\dot{\pi}^2 - \frac{1}{2}\bar{M}^2(\nabla^2\pi)^2 - \frac{M^4}{2c_*}\dot{\pi}(\nabla\pi)^2 + \frac{M^4}{8c_*^2}(\nabla\pi)^4 + \cdots\right], \tag{4.1}$$

where we have suppressed the higher order terms that arise from the $S_{1,2}$ terms giving rise to the $(\nabla^2\pi)^2$ terms.

Clearly for this to be a sensible low-energy effective theory, the cutoff has to be beneath the scales $M, \bar{M}$, which we will take to be roughly comparable. If we canonically normalize the $\dot{\pi}^2$ kinetic term, we find that the $(\nabla^2\pi)^2$ term and the cubic interaction $\dot{\pi}(\nabla\pi)^2$ term are both mass dimension 6 operators. Which term is more important at low energies? A closely related question is that of stability. The (particle physics) energy is independent of the $\dot{\pi}(\nabla\pi)^2$ term because it is linear in $\dot{\pi}$, but the quartic term $(\nabla\pi)^4$ contributes a negative-definite term to the energy density that will be larger than the positive contribution from the $(\nabla^2\pi)^2$ term provided that $\nabla\pi \gtrsim k/M$. It is therefore possible to find directions in field configuration space with arbitrarily long wavelengths in which the energy can be made more and more negative. This means that we have to address the nonlinear stability of the theory. This is most pressing at quantum level: what stops quantum fluctuations from lowering the energy and leading to vacuum instability?

The key to understanding all of these issues is to do the proper power-counting analysis for the scaling dimension of all the operators in our effective theory (see ref. [23] for a very clear discussion of these issues). In a relativistic theory, the scaling and mass dimension of couplings are the same, while this is not the case in the non-relativistic theory we are studying. Let us begin then by identifying the scaling dimensions as the energy is scaled by $E \mapsto sE$, where s is some constant, or in terms of time $t \mapsto s^{-1}t$. We determine the scaling dimension of space x and the field π by requiring that the quadratic part of the action is invariant. This then fixes

$$E \mapsto sE\,, \tag{4.2}$$

$$t \mapsto s^{-1}t\,, \tag{4.3}$$

$$x \mapsto s^{-1/2}x\,, \tag{4.4}$$

$$\pi \mapsto s^{1/4}\pi\,. \tag{4.5}$$

The scaling of x is expected since $\omega \propto k^2$, as $\omega \mapsto s\omega$ we should take $k \to s^{1/2}k$ and so $x \mapsto s^{-1/2}x$. With this scaling, it is easy to check that the leading interaction

$$\int d^3x\, dt\, M^4 \dot{\pi}(\nabla\pi)^2 \tag{4.6}$$

scales as $s^{1/4}$. It is therefore (just barely!) an irrelevant operator, becoming less and less important at low energies compared to M. All the other operators are even more irrelevant than this leading one. This implies that there is a regime of low energies and momenta, and low field amplitudes, where the expansion is under control.[2] In particular, the quantum fluctuations are not large enough to make the higher order terms compete with the quadratic terms. There are therefore no large quantum instabilities in the IR; only in the UV where the theory becomes strongly coupled is there a danger of instabilities from the higher order terms. This is however sensitive to the UV completion, and in this respect the situation is similar to QED or any other theory that gets strongly coupled in the UV. In any case, if we work with a UV cutoff Λ that is somewhat smaller than the scales

[2]This is a non-trivial result. For instance, suppose it had turned out that the leading spatial kinetic term for π was of the form $(\nabla\nabla^2\pi)^2$, then the scaling dimensions of the field would have been different ($x \mapsto s^{-1/3}x$ and $\pi \to s^0\pi$), and the leading operator $\dot{\pi}(\nabla\pi)^2$ would have scaled as $s^{-1/3}$ and been *relevant*, becoming strong and leading to a breakdown on the effective theory at low energies. Fortunately this did not happen for us. It is amusing to note that our leading operator is more irrelevant for all spacetime dimensions $d > 4$, while for $d = 3$ it is classically marginal! In this case the 1-loop beta function must be computed to determine the IR behavior of the theory.

$M, \bar{M}$, then the strongest dimensionless coupling strength in the theory is set by powers of $\lambda = \Lambda/M$. The timescale for any quantum instability, if it exists, will be exponentially small in $1/\lambda$, and can easily be longer than the age of the universe.

For completeness note also that there is an operator that is not forbidden by any symmetries, namely $(\nabla\pi)^2$. This is definitely a relevant operator, however, as we have seen, Hubble friction always drives the background to a point where the coefficient of this operator vanishes.

The fact that the theory is healthy independently of gravity is in contrast with case of the massive gravity or the DGP model. In our model there is a limit where the scalar decouples, while keeping the strength of gravity fixed. To do this, we take $M_{\rm Pl} \to \infty$ and also all matter sources $T_{\mu\nu} \to \infty$, keeping the gravitational scale $T_{\mu\nu}/M_{\rm Pl}^2$ fixed, while also keeping M fixed. In massive gravity or DGP model, there is a scalar coupled to ordinary matter with the same strength as the graviton and its effects do not vanish in this limit.

The scales M that we will be considering will turn out to be quite a bit smaller than the TeV scale, in the range $10^{-3}\,\mathrm{eV} \lesssim M \lesssim 10\,\mathrm{MeV}$, and the π sector will need to be embedded in a UV completion above this scale. However, gravity and the standard model are very weakly coupled to the π sector, so the breakdown of the π effective theory at M will not appreciably affect the standard model sector.

5. Direct coupling to standard model fields

The standard model fields need not have any direct couplings to the ghost sector (other than indirectly through gravity). However, it is interesting to consider possible direct couplings of the standard model fields to ϕ, preserving the shift symmetry on ϕ. The cutoff of the standard model sector can be much larger than the cutoff of the ghost sector as long as the couplings between them are weak enough. The leading derivative coupling of ϕ to standard model fermions is familiar from Goldstone boson interactions, and is of the form

$$\Delta\mathcal{L} = \sum_\psi \frac{c_\psi}{F}\bar{\psi}\bar{\sigma}^\mu\psi\partial_\mu\phi\,. \tag{5.1}$$

(Here we have reverted to canonical normalization for the ϕ field.) A systematic discussion of all possible Lorentz violating effects for standard model fields in flat space can be found in ref. [24, 25]. Note that these couplings can be removed by field redefinitions on the fermions $\psi \to e^{ic_\psi\phi/F}\psi$; however, if the symmetry $\psi \to e^{ic_\psi\theta}\psi$ is broken by mass terms or other couplings in the lagrangian, the interaction cannot be removed. If ψ, ψ^c are paired with a non-zero Dirac mass term $m_D\psi\psi^c$, then the vector couplings (with $c_\psi + c_{\psi^c} = 0$) can be removed, while the axial couplings

$$\Delta\mathcal{L} \sim \frac{1}{F}\bar{\Psi}\gamma^\mu\gamma^5\Psi\partial_\mu\phi \tag{5.2}$$

remain. Expanding $\phi = M^2t + \pi$, we have

$$\Delta\mathcal{L} \sim \mu\bar{\Psi}\gamma^0\gamma^5\Psi + \frac{1}{F}\bar{\Psi}\gamma^\mu\gamma^5\Psi\partial_\mu\pi\,, \qquad \text{where } \mu \equiv \frac{M^2}{F}\,. \tag{5.3}$$

JHEP05(2004)074

The first term is a Lorentz and CPT violating term, that gives rise to a different dispersion relation for particles and their antiparticles. In particular, a left helicity particle and right helicity antiparticle have dispersion relations of the form

$$\omega = \sqrt{(|p| \pm \mu)^2 + m_D^2} \tag{5.4}$$

where the $+$ sign is for the left-helicity particle and the $-$ sign is for the right-helicity antiparticle, and the signs are reversed for a right-helicity particle and left-helicity antiparticle [26, 27]. Furthermore, if the earth is moving with respect to the background in which the condensate is spatially isotropic, there is also an induced Lorentz and CPT-violating mass term of the form

$$\mu\bar{\Psi}\vec{\gamma}\gamma^5\Psi \cdot \vec{v}_{\text{earth}} \,. \tag{5.5}$$

In the non-relativistic limit, this gives rise to an interaction hamiltonian

$$\mu\vec{S} \cdot \vec{v}_{\text{earth}} \,. \tag{5.6}$$

The experimental limit on μ for the coupling to the electrons is of the order $\mu \lesssim 10^{-25}$ GeV [28], and to the proton and the neutron are $\lesssim 10^{-24}$ GeV [29, 30], assuming $|\vec{v}_{\text{earth}}| \sim 10^{-3}$. A review of Lorentz and CPT experimental tests and extensive references can be found in ref. [31].

Also strikingly, the exchange of π gives rise to a new long-range force. In the non-relativistic limit, we have a derivative coupling to spin

$$\Delta\mathcal{L} \sim \frac{1}{F}\vec{S} \cdot \nabla\pi \,, \tag{5.7}$$

As with usual Goldstone bosons, the exchange of π produces a long-range spin-dependent potential. Ordinarily for Goldstone bosons this produces a $1/r^3$ potential, but here, because of the k^4 dispersion relation instead of the usual k^2 dispersion relation, we get a $1/r$ potential:

$$V \sim \frac{M^4}{\tilde{M}^2 F^2} \frac{\vec{S}_1 \cdot \vec{S}_2 - 3(\vec{S}_1 \cdot \hat{r})(\vec{S}_2 \cdot \hat{r})}{r} \,. \tag{5.8}$$

Therefore these goldstones mediate spin-dependent inverse-square law forces! Note that we have imagined exactly static sources in deriving this force law, ignoring the retardation effects — the ω^2 piece in the denominator of the π propagator. For massless particle exchange in relativistic theories, retardation effects can be ignored as long as the sources are static on a timescale longer than the time it takes light to travel between them. In our case, however, since $\omega^2 \propto k^4$, for sources a distance r apart the static limit force law we have found is only valid on timescales longer than

$$\tau \sim \omega^{-1} \sim M\, r^2 \,. \tag{5.9}$$

We have focused here on direct couplings to the ϕ field that are linear in ϕ. These can all clearly be forbidden by a $\phi \to -\phi$ symmetry. There are however some couplings between the standard model fields and ϕ that are inevitably generated by graviton loops. The leading operators of this type have the form

$$\frac{1}{M_{\rm Pl}^4}\mathcal{O}^{\mu\nu}_{\rm SM}\partial_\mu\phi\partial_\nu\phi\,, \tag{5.10}$$

where $\mathcal{O}^{\mu\nu}_{\rm SM}$ is some dimension 4 standard model operator. Putting in the background $\dot\phi = M^2$ then gives rise to tiny dimensionless Lorentz violating effects of size $\sim M^4/M_{\rm Pl}^4$, for instance differing speeds of light for different particle species. Also, for $\mathcal{O}^{\mu\nu}_{\rm SM} = T^{\mu\nu}$, this operator can act as a coherent source for ϕ, however because of the $M_{\rm Pl}$ suppression, this is never competitive with the existing terms in the lagrangian for ϕ.

There is a large literature on Lorentz violation in the standard model [32]. The rotationally invariant effects have been thought of as arising from a non-zero value for the time component of a background vector field A_μ, $\langle A_0\rangle \neq 0$. Our model is of this type with $A_\mu \to \partial_\mu\phi$, and therefore provides a fully consistent framework to study Lorentz violation, including gravitational effects.

6. Ghost condensate coupled to gravity

In this section, we construct the effective theory of the ghost condensate coupled to gravity. We can simply do this by minimally coupling our lagrangian for ϕ to gravity, but it also instructive to see how this works directly in a "unitary gauge," where the modification of gravity is seen transparently.

To define unitary gauge, note that we can use general coordinate invariance to use ϕ as a time coordinate, i.e., we choose coordinates (t, x^i) such that

$$\phi(t,x) = t\,. \tag{6.1}$$

This is a good gauge choice for small fluctuations about a time-dependent solution such as the one we are considering. This choice of gauge eliminates ϕ as a degree of freedom, and leaves residual gauge freedom corresponding to time-dependent spatial diffeomorphisms:

$$t \mapsto t\,, \qquad x^i \mapsto x'^i(t,x)\,. \tag{6.2}$$

We can write the unitary gauge lagrangian in terms of quantities that transform simply under these residual diffeomorphism symmetry.

Let us first see this works at quadratic level in the action expanding around flat space, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Under general diffeomorphisms generated by $x^\mu \to x^\mu + \xi^\mu(x)$, we have as usual

$$\delta h_{\mu\nu} = -(\partial_\mu\xi_\nu + \partial_\nu\xi_\mu)\,. \tag{6.3}$$

Under the residual unbroken diffeomorphisms generated by ξ_i, we then have

$$\delta h_{00} = 0\,, \qquad \delta h_{0i} = -\partial_0\xi_i\,, \qquad \delta h_{ij} = -(\partial_i\xi_j + \partial_j\xi_i)\,. \tag{6.4}$$

What invariants can we use to build the action? Recall that we are assuming that flat space is a good background here, so any action must begin at quadratic order in the h's. Clearly, since h_{00} is invariant, the leading term is of the form

$$\int d^3x\, dt\, \frac{1}{8} M^4 h_{00}^2 \,. \tag{6.5}$$

As we have mentioned, because these terms do not preserve the full diffeomorphism invariance of the theory, we are really introducing an additional degree of freedom. To see this explicitly, it is convenient to re-introduce the field π that restores the full diffeomorphism symmetry of the theory — we can achieve this simply by performing a broken ξ^0 diffeomorphism and promoting $\xi^0 = \pi$ to a field. Then

$$h_{00} \to h_{00} - 2\partial_0 \pi \,, \qquad h_{0i} \to h_{0i} - \partial_i \pi \,, \qquad h_{ij} \to h_{ij} \,, \tag{6.6}$$

and we see that the $M^4 h_{00}^2$ action has generated a time kinetic term $\frac{1}{2} M^4 \dot{\pi}^2$ but no spatial kinetic term for π. In fact, it is easy to see that only adding the $M^4 h_{00}^2$ term to the action, we have not modified GR at all (at least classically), since this is a partial gauge-fixing term that fixes to $h_{00} = 0$ gauge. We must therefore go to higher order, to make π a dynamical field, and see a real modification of GR rather than just a gauge fixing.[3]

In this language, it is easy to see why we do not get a $(\nabla\pi)^2$ kinetic term. We could get this from terms of the form h_{0i}^2 in the action, since upon introducing the π we have $h_{0i} \to h_{0i} - \partial_i \pi$. However, h_{0i}^2 is not invariant under the residual diffeomorphisms! There are invariants that can be constructed, but they involve higher derivatives, for instance

$$\begin{aligned} K_{ij} &= \frac{1}{2}(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}) \\ &\to \frac{1}{2}(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j} + \partial_i \partial_j \pi) \end{aligned} \tag{6.7}$$

is invariant (and is the linearized extrinsic curvature of constant time surfaces in the theory). The leading terms at quadratic level in the effective theory are of the form

$$S = \int d^3x\, dt \left[-\frac{1}{2} \tilde{M}^2 K_{ii}^2 - \frac{1}{2} \tilde{M}'^2 K_{ij} K_{ij} \right]. \tag{6.8}$$

Terms linear in K can be forbidden by time-reversal invariance. If we do add for example $h_{00} K \to \dot{\pi} \nabla^2 \pi + \cdots$, it contributes a term ωk^2 to the π dispersion relation. However, this does not qualitatively change the physics as $\omega \sim k^2$ still holds. Terms linear in $\dot{\pi}$ do not contribute to the hamiltonian and hence do not affect stability either.

Upon re-introducing π, (6.8) gives us the k^4 spatial kinetic terms,

$$S = \int d^3x\, dt \left[-\frac{1}{2} \bar{M}^2 (\nabla^2 \pi)^2 + \cdots \right], \qquad \bar{M}^2 \equiv (\tilde{M}^2 + \tilde{M}'^2) \,, \tag{6.9}$$

[3] The h_{00}^2 term does not fix time reparameterization globally. In particular, static solutions are not unique. The correct procedure is to define static solutions as the limit of time-dependent solutions, as will be done in specific examples below.

so we see that the k^4 spatial kinetic terms are a direct consequence of the assumption that flat space is a good background, and the unbroken residual spatial diffeomorphisms of the theory.

We can repeat the same analysis for a de Sitter background, the metric is of the form

$$ds^2 = (1 + h_{00})dt^2 - e^{2Ht}dx^i dx^j(\delta_{ij} - h_{ij}) + 2h_{0i}dtdx^i\,, \tag{6.10}$$

and the linearized diffeomorphism transformations are

$$\delta h_{00} = -2\partial_0\xi_0\,, \qquad \delta h_{0i} = -\partial_i\xi_0 - e^{2Ht}\partial_0\xi_i\,, \qquad \delta h_{ij} = 2H\xi_0\delta_{ij} - \partial_i\xi_j - \partial_j\xi_i\,, \tag{6.11}$$

where $\xi_\mu = \eta_{\mu\nu}\xi^\nu$. As before, we can restore full diffeomorphism invariance by promoting ξ_0 to a dynamical field π. Then the only invariants under linearized diffeomorphisms are

$$h_{00} - 2\dot\pi\,, \qquad \text{and} \qquad \dot h_{ij} - e^{-2Ht}(\partial_i h_{0j} + \partial_j h_{0i} - \partial_i\partial_j\pi) + 2\delta_{ij}H\dot\pi\,. \tag{6.12}$$

As before, we cannot write an invariant term that contains a $(\nabla\pi)^2$ kinetic term.

Let us now proceed to the systematic construction of the effective theory at non-linear level. First, consider the scalar quantity

$$X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\,. \tag{6.13}$$

In unitary gauge,

$$X \to g^{00}\,, \tag{6.14}$$

and we see that in unitary gauge the lagrangian can be an arbitrary function of g^{00}. Writing the lagrangian in terms of X is useful because it is obvious how to generalize to an arbitrary gauge.

We can also define the unit vector perpendicular to the surfaces of constant ϕ:

$$n_\mu = \frac{\partial_\mu\phi}{\sqrt{X}} \to \frac{\delta_\mu{}^0}{\sqrt{g^{00}}}\,. \tag{6.15}$$

This can be used to project any tensor into tensors with all indices parallel to the surfaces of constant ϕ. For example, the induced metric is simply

$$\gamma_{\mu\nu} = -(g_{\mu\nu} - n_\mu n_\nu)\,, \tag{6.16}$$

which satisfies $n^\mu\gamma_{\mu\nu} = 0$, and is therefore a metric on the surfaces of constant ϕ. We can use this to write other tensors such as the intrinsic curvature $R(\gamma)$ and the extrinsic curvature

$$K_{\mu\nu} = \gamma_\mu{}^\rho\nabla_\rho n_\nu\,, \tag{6.17}$$

which is a symmetric tensor satisfying $n^\mu K_{\mu\nu} = 0$.

Since the residual diffeomorphisms depend on time, ∂_0 is not a covariant derivative. The covariant time derivative on tensors with all indices parallel to surfaces of constant ϕ is

$$D^\phi T^{\cdots}{}_{\cdots} = \frac{1}{\sqrt{X}}(n^\mu\nabla_\mu T^{\cdots}{}_{\cdots})^{\|}\,, \tag{6.18}$$

where '$\|$' denotes the projection onto surfaces of constant ϕ using n_μ. (The normalization is dictated by the fact that $n^\mu \partial_\mu = \partial^0$ in unitary gauge.)

The most general effective lagrangian invariant under spatial diffeomorphisms in unitary gauge is therefore

$$\mathcal{L} = \sqrt{\gamma}\left[F(X) + \frac{1}{2}Q_1(X)K^2 + \frac{1}{2}Q_2(X)K^{ij}K_{ij} + \cdots + \text{terms involving time derivatives}\right]. \tag{6.19}$$

Here the ij indices are raised and lowered with γ_{ij}, γ^{ij}. Again we have assumed the time-reversal symmetry forbidding the terms linear in K. Note also that at this order we could have written down a term involving the intrinsic curvature $R^{(3)}$, however, the full four-dimensional curvature $R^{(4)}$ is given by $R^{(4)} = R^{(3)} + K_{ij}^2 - K^2$, so we can always write a term linear in $R^{(3)}$ as a usual Einstein-Hilbert term plus terms proportional to K^2, K_{ij}^2.

Note that we can easily restore the dependence on the ghost fluctuations π by using the general formulas above. Using the relation $\sqrt{-g} = \sqrt{\gamma}/\sqrt{X}$, we see that

$$F(X) = M^4 \frac{P(X)}{\sqrt{X}}. \tag{6.20}$$

Let us now consider the expansion about flat space in unitary gauge. The terms that do not involve derivatives in the unitary gauge lagrangian are

$$\mathcal{L} = \frac{\sqrt{\gamma}M^4 P(X)}{\sqrt{X}}. \tag{6.21}$$

We would like to have a solution where $g_{\mu\nu} = \eta_{\mu\nu}$, so $X = \eta^{00} = 1$. It is easy to see that this requires $P(1) = 0$ to avoid tadpoles for fluctuations of the spatial metric, and $P'(1) = 0$ to avoid tadpoles for g^{00}. The fact that $P(1) = 0$ can be interpreted as the condition that there is no cosmological constant (see eq. (2.11)). The fact that $P'(1) = 0$ implies that the ghost fluctuations have no k^2 term in their dispersion relation as we have seen, and the leading terms are the k^4 ones (see eq. (2.6)).

Summarizing, the leading low-energy effective action is, in unitary gauge,

$$S = \int d^3x\, dt \sqrt{\gamma}\left[\frac{1}{8}M^4(X-1)^2 - \frac{1}{2}\tilde{M}^2 K^2 - \frac{1}{2}\tilde{M}'^2 K_{ij}K^{ij} + \cdots\right]. \tag{6.22}$$

7. Infrared modification of gravity

We now consider the modification of gravity in the ghost condensate, working at linearized level. At this order, the only effect comes from the mixing between π and gravity, and so it suffices to only consider scalar perturbations of the metric. A general scalar perturbation can be parameterized by

$$h_{00} = 2\Phi', \qquad h_{0i} = \partial_i \chi, \qquad h_{ij} = 2\delta_{ij}\Psi' + \partial_i\partial_j\eta. \tag{7.1}$$

Using the the diffeomorphisms generated by $\xi_i = \frac{1}{2}\partial_i\eta$, $\xi_0 = \chi - \frac{1}{2}\partial_0\eta$, we can bring the metric to the longitudinal gauge,

$$h_{00} = 2\Phi, \qquad h_{0i} = 0, \qquad h_{ij} = 2\Psi\delta_{ij}. \tag{7.2}$$

JHEP05(2004)074

We can analyze the physics at the level of the quadratic action (or equivalently the linearized equation of motion). Let us begin by working at the level of the action. The Einstein-Hilbert action in this gauge is,

$$S_{\rm EH} = M_{\rm Pl}^2 \left[-3(\partial_0 \Psi)^2 - 2\nabla\Psi\cdot\nabla\Phi + (\nabla\Psi)^2 \right], \qquad M_{\rm Pl}^2 \equiv \frac{1}{8\pi G}\,. \tag{7.3}$$

Note that despite the appearance of the time derivative on Ψ in this action, neither Φ nor Ψ are dynamical fields. Indeed it is easy to go to momentum space and diagonalize the 2×2 kinetic matrix mixing Φ, Ψ; the zeros of the determinant of this matrix give the dispersion relation which is just $k^4 = 0$. This is in keeping with the fact that the only propagating fields in gravity are the two polarizations of the spin-two massless gravitons. Note also that in the newtonian limit, $\omega^2 \ll k^2$, so the action reduces to $-2\nabla\Psi\cdot\nabla\Phi + (\nabla\Psi)^2$, and the Ψ equation of motion then fixes $\Psi = \Phi$.

We now include the ghost condensate. We again work in the newtonian limit where $\omega^2 \ll k^2$. Then at quadratic level the lagrangian for π becomes

$$\mathcal{L} = \frac{1}{2}M^4(\Phi - \dot\pi)^2 - \frac{1}{2}\bar M^2(\nabla^2\pi)^2\,. \tag{7.4}$$

We can then put Ψ to its equation of motion $\Psi = \Phi$, and go to canonical normalization $\Phi = \Phi_c/(\sqrt{2}M_{\rm Pl}), \pi = \pi_c/M^2$, to find the lagrangian

$$\mathcal{L} = -\frac{1}{2}(\nabla\Phi_c)^2 + \frac{1}{2}\left(\frac{M^2}{\sqrt{2}M_{\rm Pl}}\Phi_c - \dot\pi_c\right)^2 - \frac{\bar M^2}{2M^4}(\nabla^2\pi_c)^2\,. \tag{7.5}$$

We can see that there are small mixing termqs between Φ_c and $\dot\pi_c$. Going to momentum space, the lagrangian in the (π_c, Φ_c) basis is

$$\mathcal{L} = \frac{1}{2}(\pi_c\, \Phi_c)\begin{pmatrix} \omega^2 - \alpha^2 k^4/M^2 & -im\omega \\ im\omega & -k^2 + m^2 \end{pmatrix}\begin{pmatrix} \pi_c \\ \Phi_c \end{pmatrix}, \tag{7.6}$$

where we have defined

$$m \equiv \frac{M^2}{\sqrt{2}M_{\rm Pl}}, \qquad \alpha^2 \equiv \frac{\bar M^2}{M^2}\,. \tag{7.7}$$

We see that the π dispersion relation is modified from $\omega^2 = \alpha^2 k^4/M^2$ due to mixing with gravity. The dispersion relation is found by setting the determinant of the 2×2 matrix above to zero, which gives

$$\omega^2 = \alpha^2\frac{k^4}{M^2} - \alpha^2\frac{m^2}{M^2}k^2 = \alpha^2\frac{k^4}{M^2} - \frac{\alpha^2 M^2}{2M_{\rm Pl}^2}k^2\,. \tag{7.8}$$

For very low momenta $k < m$, we find $\omega^2 < 0$, signaling an instability. This is the analog for our fluid of the usual Jeans instability for ordinary fluids; for a fluid of mean density ρ and pressure p, the usual Jeans instability shows up as a modification of the dispersion of the form[4]

$$\omega^2 = \frac{p}{\rho}k^2 - \omega_J^2\,, \qquad \text{where } \omega_J^2 = \frac{\rho}{2M_{\rm Pl}^2}\,. \tag{7.9}$$

[4]Strictly speaking, the dispersion relation eq. (7.9) is not correct, because in ordinary gravity flat space is not a solution when $\rho \neq 0$ (the 'Jeans swindle'). In the ghost condensate, flat space *is* a solution, so the dispersion relation eq. (7.8) is not a swindle.

JHEP05(2004)074

In our case, the largest imaginary magnitude of ω for the instability is

$$\omega_{\rm inst} = i\frac{\alpha M^3}{4M_{\rm Pl}^2} \equiv i\Gamma\,. \tag{7.10}$$

Just as the usual Jeans instability is removed by Hubble friction in an expanding universe, we will see in the next section that for our case that in a de Sitter background with $H > \Gamma$, this instability is also removed.

As we will now see, Γ^{-1} is also the timescale over which modifications of gravity take place, at a length scale of order m^{-1}. To see this, we can look at the modification of the Φ_c propagator. If we immediately go to the static limit $\omega \to 0$, we see that the Φ_c propagator is simply modified from $1/k^2 \to 1/(k^2 - m^2)$ (remembering that k is the 3-momentum). This looks like a *negative* mass squared for Φ_c (the sign is dictated by the requirement of a healthy π time kinetic term), and so gives rise to an oscillatory $\sin(mr)/r$ or $\cos(mr)/r$ modification of the newtonian potential, rather than the familiar e^{-mr}/r Yukawa form. However, as we already saw for the spin-dependent force mediated by π exchange, because the π's do not move at the speed of light but instead slow down in their propagation at larger length scales, retardation effects are very important, and we cannot immediately go to the static $\omega \to 0$ limit. It is straightforward to find the $\langle\Phi_c\Phi_c\rangle$ propagator by inverting the 2×2 kinetic matrix, yielding

$$\left[1 - \frac{\alpha^2 m^2 k^2}{M^2\omega^2 - \alpha^2 k^4 + \alpha^2 m^2 k^2}\right] \times \left(-\frac{1}{k^2}\right). \tag{7.11}$$

The factor in the brackets represents a change in the newtonian potential. To get an $O(1)$ modification of gravity, we must have

$$\omega^2 - \frac{\alpha^2}{M^2}k^4 \lesssim \alpha^2\frac{m^2}{M^2}k^2\,. \tag{7.12}$$

This tells us that we have to be close to "on-shell" for the π excitations; directly in terms of ω we then have

$$\left|\omega - \frac{\alpha k^2}{M}\right| \lesssim \frac{\alpha m^2}{M} \sim \Gamma\,. \tag{7.13}$$

So we see that the width of the region in ω is small, set by $\Delta\omega \sim \Gamma$, and therefore we must wait for a long timescale $t_c \sim \Gamma^{-1}$ to see any modifications of gravity. It is also easy to see that the natural length scale of the modification is $r_c \sim m^{-1}$. Indeed, if we work with dimensionless variables $\hat{\omega}, \hat{k}$ defined by $\omega = 2\Gamma\hat{\omega}$, $k = m\hat{k}$, we see that the modulation factor for the newtonian potential is simply

$$\left[1 - \frac{\hat{k}^2}{\hat{\omega}^2 - \hat{k}^4 + \hat{k}^2}\right] \tag{7.14}$$

and so modifications occur for $\hat{\omega}, \hat{k} \sim O(1)$. We could have also seen that r_c, t_c are the natural length and time scales for modification also directly from the action. By further defining $\hat{\Phi} = m\Phi_c$ and $\hat{\pi} = \Gamma^{-1}\pi_c$, the action can be seen to be $O(1)$ in all hatted variables.

We therefore conclude that modifications of gravity take place at a distance r_c and time t_c given by

$$r_c \sim \frac{M_{\rm Pl}}{M^2}\,, \qquad t_c \sim \frac{M_{\rm Pl}^2}{M^3\alpha}\,. \tag{7.15}$$

The fact that $t_c \gg r_c$ makes sense from a number of points of view. Note that the scale r_c is completely determined by the $M^4 h_{00}^2$ part of the lagrangian. However, as we have already remarked, with only this term, classically we have not modified gravity at all, only fixed to $h_{00} = 0$ gauge. Only with the other terms (which lead to the k^4 spatial kinetic terms for π) do we see that there is a genuine modification of gravity — to see any change in the potential we then have to wait until both terms become important, and the timescale is then larger than what one might expect from the coefficient of the h_{00}^2 term alone.

Let us see how the modification works explicitly in an example, by calculating the effective gravitational potential that is felt by a test mass, outside a source which turns on at the origin at time $t = 0$, i.e., a source

$$\rho_m(r,t) = \delta^3(r)\theta(t)\,. \tag{7.16}$$

Of course energy is really conserved here; this can be thought of as an approximation to a situation where a cloud of dust quickly collapses to a point-like mass at the origin. We are interested in the gravitational force that a test particle feels a distance r away, as a function of time $t > 0$. Given the Φ_c propagator we have calculated, it is easy to Fourier transform back to position space to find

$$\Phi(r,t) = -\frac{G}{r}\left[1 + I(r,t)\right], \tag{7.17}$$

where $I(r,t)$ is a spatial Fourier integral over the momentum k. Introducing the dimensionless variables

$$u = \frac{k}{m}, \quad R = mr\,, \qquad T = \frac{\alpha M^3}{2M_{\rm Pl}^2} = 2\Gamma t\,, \tag{7.18}$$

we have

$$I(r,t) = \frac{2}{\pi}\left\{\int_0^1 du \frac{\sin(uR)}{(u^3-u)}\left(1 - \cosh(Tu\sqrt{1-u^2})\right) + \right.$$
$$\left. + \int_1^\infty du \frac{\sin(uR)}{(u^3-u)}\left(1 - \cos(Tu\sqrt{u^2-1})\right)\right\}. \tag{7.19}$$

For times $t \ll t_c\,(T \ll 1)$, $I(r,t) \to 0$ and there is no modification of the standard newtonian potential. However, for $t \gtrsim t_c\,(T \gtrsim 1)$, there are modifications. The presence of the exponentially growing $\cosh(Tu\sqrt{1-u^2})$ term in the first integrand is a reflection of the Jeans-type instability we found in flat space. For large T, therefore, the first integrand will dominate the integral for $I(r,t)$. The exponential in the cosh is maximized for $u = 1/\sqrt{2}$;

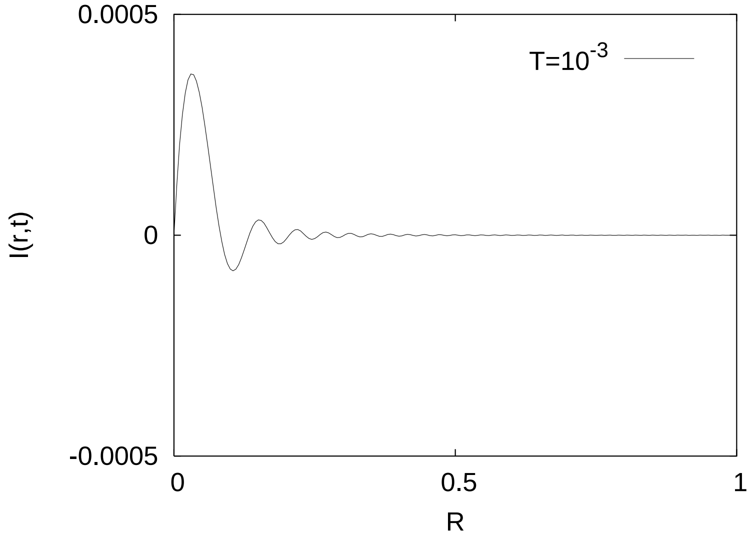

Figure 5: The newtonian potential is $V(r) = -\frac{G}{r}(1 + I(r = \frac{R}{m}, t = \frac{T}{2\Gamma}))$. The function $I(r, t = .001(2\Gamma)^{-1})$ is plotted here.

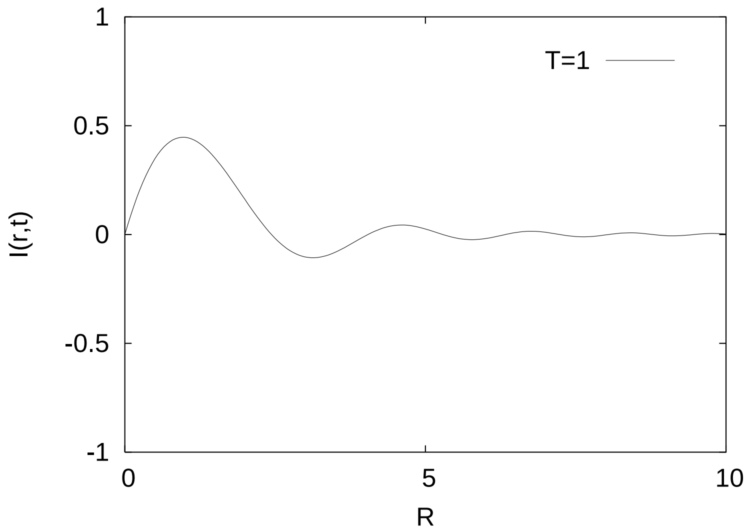

Figure 6: Same as the previous figure except for $t = (2\Gamma)^{-1}$. Note the axes have been rescaled.

approximating the integrand by a gaussian around this point and performing the resulting gaussian integration then gives us an excellent approximation for $I(r,t)$:

$$I(r,t) \simeq \frac{2}{\sqrt{\pi T}} \exp\left(-\frac{R^2}{8T} + \frac{T}{2}\right) \sin\left(\frac{R}{\sqrt{2}}\right) . \tag{7.20}$$

So, for $R \ll T$ $(r/r_c \ll t/t_c)$, we indeed find an oscillatory behavior for the newtonian potential, which is growing exponentially in time as $\exp(T/2)$ as a consequence of the instability! However, for $R \gg T$, the correction vanishes; it takes a time of order $t = t_c \times (r/r_c)$ for any appreciable changes to happen at distance r. The exact behavior of this newtonian potential at various time is shown in figures 5–7.

JHEP05(2004)074

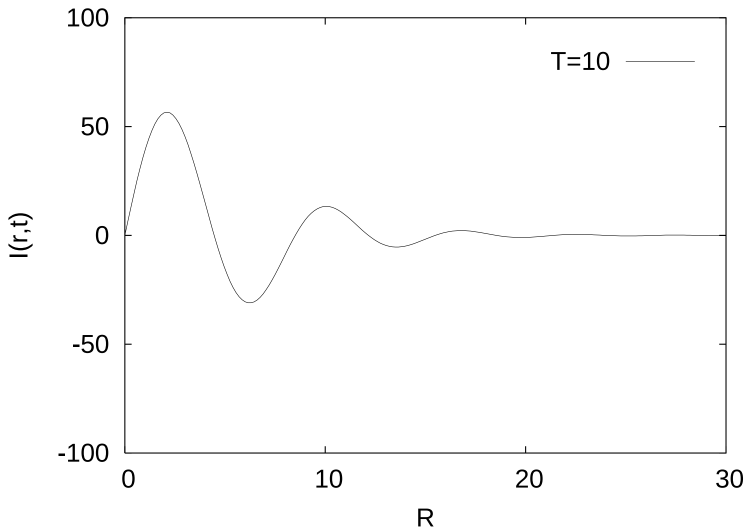

Figure 7: Same as the previous figure this time for $t = 5\Gamma^{-1}$. Note again that the axes are rescaled and the correction at linear level has become (exponentially) larger in amplitude. This is due to the Jeans instability of the fluid in flat space.

Note that for $M \sim 10^{-3}\,\mathrm{eV}$, which could drive the acceleration of the universe today, $r_c \sim H_0^{-1}$ is about the size of the universe, while $t_c \gg H_0^{-1}$ is much larger than the age of the universe! Thus, modifications of the newtonian potential will occur only at much later time than the present age of the universe. On the other hand, for $M \sim 10\,\mathrm{MeV}$, we can have $t_c \sim H_0^{-1}$ comparable to the present age of the universe, and these oscillatory modifications of gravity take place on a distance scale of $\sim 1000\,\mathrm{km}$!

We close this section by making some brief comments on the modifications of gravity at non-linear level. In backgrounds with large gravitational fields such as a neutron star, large redshift factors will make $\dot{\phi}$ vary by $O(1)$, and we might wonder whether this leads to large deviations from GR. Recall that there is a decoupling limit in which all the effects of the ghost condensate on the matter sector vanish: $M_{\rm Pl}$ and all matter source energy densities $\rho_m \to \infty$, while keeping the usual gravitational scales $\rho_m/M_{\rm Pl}^2$ fixed, and also M fixed. In this limit the π's have no back-reaction at all on the metric. Beyond this, it is easy to see that with only the $P(X)$ term in the lagrangian, if we begin with initial data with $P'(X = c_*^2) = 0$ (as we know we are driven to by Hubble friction), then $X = c_*^2$ for all time. Thus, in a non-trivial gravitational potential, ϕ adjusts itself so that $X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is fixed and there is no modification of GR. Only with the additional terms in the action (that give rise to the k^4 spatial kinetic terms) do we get a deviation from $X = c_*^2$. As we saw already at linearized level, however, this effect takes time to build up. At any rate, due to the existence of the healthy decoupling limit, there is some limit on M which will be in agreement with all tests of GR. Clearly $M \sim 10^{-3}\,\mathrm{eV}$ will be fine, since in this case even the most naive estimate of the scale of modification is $M^2/M_{\rm Pl} \sim H_0$. A more detailed investigation is needed to find the precise bounds near the much larger values of $M \sim 10\,\mathrm{MeV}$, that lead to deviations from the linearized newtonian gravity at timescales of order the current age of the universe at distances of order ~ 1000 km. It is possible that nonlinear solutions evolve to states outside the regime of validity of the effective theory, but the same arguments lead us to expect that this takes very long times.

8. The ghost condensate in de Sitter space

In the previous section we studied the ghost condensation and modification of gravity in a Minkowski background, and we saw that there is a Jeans-like instability. As in the case of the Jeans instability for ordinary matter, we expect that it will go away in an expanding universe if the expansion rate exceeds the growth rate of the instability. In this section, we study the ghost condensation and modification of gravity in such an expanding de Sitter background. This is relevant since the current expansion of the universe is accelerating and we are approaching a de Sitter space. One could also consider the possibility that ghost condensation is responsible for inflation in the early universe. In that case, the fluctuations of ghost condensation in de Sitter space can be relevant for generating the density perturbations that seed structure formation in the universe.

It is more convenient to do the analysis using the linearized equations of motion, so for ease of comparison with our results from the last section, let us begin by writing the linearized equations of motion in flat space. In longitudinal gauge, the Einstein tensor in flat space is

$$G_{00} = 2\nabla^2\Psi, \tag{8.1}$$

$$G_{0i} = 2\partial_0\partial_i\Psi, \tag{8.2}$$

$$G_{ij} = \delta_{ij}(2\partial_0^2\Psi + \nabla^2(\Phi - \Psi)) + \partial_i\partial_j(\Psi - \Phi)\,. \tag{8.3}$$

and the equations of motion are

$$G_{00} + \frac{1}{2}M^4(h_{00} - 2\partial_0\pi) = 0\,, \tag{8.4}$$

$$-G_{0i} - \alpha^2 M^2(\partial_i K) = 0\,, \tag{8.5}$$

$$G_{ij} + \alpha^2 M^2 \partial_0 K \delta_{ij} = 0\,, \tag{8.6}$$

$$-\partial_0(\partial_0\pi - \frac{1}{2}h_{00}) - \frac{\alpha^2\nabla^2}{M^2}K = 0\,, \tag{8.7}$$

where we have kept only h_{00}^2 and K^2 terms and set $M_{\rm Pl}^2 = 1/(8\pi G) = 1$ for convenience.

From the third equation, for $i \neq j$, we find that $\Phi = \Psi$. Further dropping subleading terms for $M/M_{\rm Pl} \ll 1$, we find

$$2\nabla^2\Phi + M^4\Phi - M^4\partial_0\pi = 0\,, \tag{8.8}$$

$$-2\partial_0\partial_i\Phi - \alpha^2M^2\partial_i\nabla^2\pi = 0\,, \tag{8.9}$$

$$2\partial_0^2\Phi + \alpha^2M^2\partial_0\nabla^2\pi = 0\,, \tag{8.10}$$

$$\partial_0^2\pi - \partial_0\Phi + \frac{\alpha^2\nabla^4}{M^2}\pi = 0\,. \tag{8.11}$$

The second and third equations in the above both imply that

$$\partial_0\Phi = -\frac{1}{2}\alpha^2M^2\nabla^2\pi\,. \tag{8.12}$$

Inserting this into the π equation of motion yields

$$\partial_0^2\pi + \frac{1}{2}\alpha^2 M^2\nabla^2\pi + \alpha^2\frac{\nabla^4}{M^2}\pi = 0\,. \tag{8.13}$$

From here we can see the modified dispersion relation for π, and together with the equation for $\nabla^2\Phi$, the modified propagator for Φ can be derived.

We repeat this exercise in a de Sitter background. As before, we can use diffeomorphisms to bring the metric to the following form,

$$ds^2 = (1+2\Phi)dt^2 - a(t)^2(1-2\Psi)d\mathbf{x}^2\,, \qquad a(t) = e^{Ht}\,. \tag{8.14}$$

The equations of motion (8.8)–(8.11) are modified to

$$-6H\partial_0\Psi + 2\frac{\nabla^2}{a^2}\Psi - 6H^2\Phi + M^4(\Phi - \partial_0\pi) = 0\,, \tag{8.15}$$

$$-\partial_i\left(2H\Phi + 2\partial_0\Psi\right) - \alpha^2 M^2\partial_i\left(3\partial_0\Psi + \frac{\nabla^2}{a^2}\pi\right) = 0\,, \tag{8.16}$$

$$\begin{aligned}\delta_{ij}&\left[2H\partial_0(\Phi+3\Psi) + 2\partial_0^2\Psi + 6H^2(\Phi+\Psi) + \frac{\nabla^2}{a^2}(\Phi-\Psi)\right] - \\ -\frac{\partial_i\partial_j}{a^2}(\Phi-\Psi) - \delta_{ij}&\left[6H^2\Psi - \alpha^2M^2(\partial_0+3H)\left(3\partial_0\Psi + \frac{\nabla^2}{a^2}\pi\right)\right] = 0\,,\end{aligned} \tag{8.17}$$

$$(\partial_0 + 3H)(\Phi - \partial_0\pi) - \frac{\alpha^2}{M^2}\left(3\partial_0\Psi + \frac{\nabla^2}{a^2}\pi\right) = 0\,. \tag{8.18}$$

We consider that the Hubble expansion parameter lies in the interesting range,

$$\Gamma\left(=\frac{\alpha M^3}{4M_{\rm Pl}^2}\right) \lesssim H \ll m\left(=\frac{M^2}{\sqrt{2}M_{\rm Pl}}\right) \ll M\,. \tag{8.19}$$

For $H \ll \Gamma$ the space-time is effectively flat. On the other hand, if $H > m$, the length scale of the potential modulation is outside the horizon and the timescale for π to modulate gravity is way longer than H^{-1}, so the interesting modification of gravity could not have happened.

Again, from the third equation with $i \neq j$, we have $\Phi = \Psi$. Eliminating Φ and Ψ from the above equations and neglecting higher order terms, we obtain the equation of motion for the π field,

$$\left(\frac{M^4}{2} + \frac{\nabla^2}{a^2}\right)\partial_0^2\pi + H\left(\frac{M^4}{2} + 3\frac{\nabla^2}{a^2}\right)\partial_0\pi + \frac{\alpha^2}{M^2}\frac{\nabla^2}{a^2}\left(\frac{M^4}{2} + \frac{\nabla^2}{a^2}\right)^2\pi = 0\,. \tag{8.20}$$

By Fourier transforming the spatial dependence, $\nabla^2 \to -k^2$, the equation of each mode k can be written as

$$\partial_0^2\pi + H\left(\frac{m^2 - 3\frac{k^2}{a^2}}{m^2 - \frac{k^2}{a^2}}\right)\partial_0\pi - \frac{4\Gamma^2}{m^4}\frac{k^2}{a^2}\left(m^2 - \frac{k^2}{a^2}\right)\pi = 0\,. \tag{8.21}$$

The $\partial_0\pi$ term provides the friction (anti-friction) for modes with $k/a > m$ or $k/a < m/\sqrt{3}$ ($m/\sqrt{3} < k/a < m$). The redshift will eventually bring each mode to the regime $k/a \ll$

$m/\sqrt{3}$ and the friction term becomes simply $H\partial_0\pi$. For $H \gtrsim \Gamma$, the friction term dominates and the instability disappears just like the usual Jeans instability.

To study the modification of gravity due to the ghost condensate, we should examine the gravitational potential by eliminating π instead. The equation for the gravitational potential Φ without additional source is

$$\left(\partial_0^2 + 3H\partial_0 + 2H^2\right)\Phi + \frac{\alpha^2}{M^2}\left(\frac{\nabla^2}{a^2}\right)^2\Phi + \frac{\alpha^2 M^2}{2}\frac{\nabla^2}{a^2}\Phi = 0\,, \tag{8.22}$$

or for each momentum mode k,

$$\left(\partial_0^2 + 3H\partial_0 + 2H^2\right)\Phi + \frac{4\Gamma^2}{m^4}\left(\frac{k^4}{a^4}\right)\Phi - \frac{4\Gamma^2}{m^2}\left(\frac{k^2}{a^2}\right)\Phi = 0\,. \tag{8.23}$$

They are equivalent to eqs. (8.20), (8.21). Absence of instability for $H \gtrsim \Gamma$ can also be shown with the Φ equation. At a large distance ($\gg m^{-1}$) from a local lump of excitations, one can neglect the last two terms of the equation. The solutions at a fixed comoving distance r are simply

$$\Phi(r,t) \sim b_1(r)\, e^{-Ht} + b_2(r)\, e^{-2Ht}\,. \tag{8.24}$$

They decay at least as fast as the redshifting so there is no growing potential.

The equation of the gravitational potential with ordinary matter source can be similarly derived. It is convenient to decompose Φ into two parts as

$$\Phi = \Phi_{\rm GR} + \Phi_{\rm mod}\,, \tag{8.25}$$

where each part satisfies

$$\frac{\nabla^2}{a^2}\Phi_{\rm GR} = \frac{\rho_{\rm eff}}{2}, \quad \rho_{\rm eff} = \rho - 4\nabla^2\tilde{p}\,, \tag{8.26}$$

and

$$\left(\partial_0^2 + 3H\partial_0 + 2H^2\right)\Phi_{\rm mod} + \frac{4\Gamma^2}{m^4}\left(\frac{\nabla^2}{a^2}\right)^2\Phi_{\rm mod} + \frac{4\Gamma^2}{m^2}\frac{\nabla^2}{a^2}\Phi_{\rm mod} = -\frac{4\Gamma^2}{m^2}\frac{\nabla^2}{a^2}\Phi_{\rm GR}\,, \tag{8.27}$$

respectively, for the general scalar-type matter source

$$T_{00} = \rho\,, \qquad T_{0i} = \partial_i q\,, \qquad a^{-2}T_{ij} = p\delta_{ij} + \left(2\nabla_i\nabla_j - \frac{2}{3}\delta_{ij}\nabla^2\right)\tilde{p}\,. \tag{8.28}$$

To derive the above equations we have used the conservation equations

$$\begin{aligned} \dot{\rho} + 3H\rho + 3Hp - \frac{\nabla^2}{a^2}q &= 0\,, \\ \dot{q} + 3Hq - \frac{4}{3}\nabla^2\tilde{p} - p &= 0\,. \end{aligned} \tag{8.29}$$

Note that $\Phi_{\rm GR}$ is exactly the gravitational potential in general relativity and, thus, $\Phi_{\rm mod}$ represents the modulation of gravity due to the ghost condensation. Equation (8.27) shows that the timescale of the modulation is indeed Γ^{-1} for the length scale m^{-1}.

JHEP05(2004)074

To obtain the potential as a function of the physical distance, we can switch to the physical coordinates $X = a(t)x$, then the equation for $\Phi_{\rm mod}$ becomes

$$(\partial_0 + H\,\mathbf{X}\cdot\nabla_X)^2\Phi_{\rm mod} + 3H(\partial_0 + H\,\mathbf{X}\cdot\nabla_X)\Phi_{\rm mod} + 2H^2\,\Phi_{\rm mod} + \\ + \frac{4\Gamma^2}{m^4}\left(\nabla_X^2\right)^2\Phi_{\rm mod} + \frac{4\Gamma^2}{m^2}\nabla_X^2\Phi_{\rm mod} = -\frac{2\Gamma^2}{m^2}\rho_{\rm eff} \qquad (8.30)$$

The modulation of the gravitational potential from a source also starts at the time scale Γ^{-1} as the π field slowly reacts to the gravitational potential. If we are interested in the potential at very late time, we can look for time independent solutions for Φ by setting $\partial_0 = 0$ as there is no instability in this case. Assuming spherical symmetry, the time independent equation is

$$(\sigma^2\partial_\sigma^2 + 4\sigma\partial_\sigma + 2)\Phi_{\rm mod} + \frac{4\Gamma^2}{H^2}\left(\partial_\sigma^4 + \frac{4}{\sigma}\partial_\sigma^3\right)\Phi_{\rm mod} + \frac{4\Gamma^2}{H^2}\left(\partial_\sigma^2 + \frac{2}{\sigma}\partial_\sigma\right)\Phi_{\rm mod} = -\frac{2\Gamma^2}{H^2}\frac{\rho_{\rm eff}}{m^2}\,, \qquad (8.31)$$

where $\sigma \equiv m|X|$.

We first consider the solutions without matter source and, thus, without $\Phi_{\rm GR}$. The regularity at the origin $\sigma = 0$ requires that all odd-order derivatives of $\Phi_{\rm mod}$ vanish at $\sigma = 0$. Hence, there are two independent solutions. The two independent solutions are shown in figure 8 for $H/\Gamma = 2$, where we plot the potential multiplied by the distance from the origin. One can see that apart from the oscillatory modulation, the potential is proportional to $1/R$ at large R. This can be interpreted as arising from an effective mass produced by a lump of π excitations around the origin. The expansion of the universe keeps these π excitations stationary ($\dot\pi \neq 0$), resulting in a static potential. Any linear combination of these two configurations represents a possible gravitational potential produced by π without matter source, and the effective mass can be either positive or negative which corresponds to a gravitating or an antigravitating object respectively.

A time-independent solution of the potential with a regular matter source can also be obtained similarly. A solution with $\rho_{\rm eff}/m^2 = 0.1e^{-1.0\sigma^2}$ for $H/\Gamma = 2$ is shown in figure 9. Again we see the interesting modulation of the $1/R$ potential. The mass $-R\Phi$ [33] seen by an observer far away ($m^{-1} \ll R \ll H^{-1}$) is not necessarily the same as the mass of the matter source due to the screening or anti-screening of the π field. A general time-independent solution with the same matter source is given by the sum of the special solution shown in figure 9 and an arbitrary linear combination of the two source-free solutions shown in figure 8. The coefficients of the source-free solutions cannot be determined by the time independent equation (8.31) but should be determined by dynamical considerations.

This situation is in stark contrast to the corresponding situation in general relativity. In general relativity, a spherical symmetric spacetime outside matter source is locally static and, thus, locally a Schwarzschild geometry. This fact, known as the Birkhoff theorem [34], makes it possible to characterize the geometry outside the source just by a mass parameter. In general relativity, this is true even if the geometry inside the matter source is completely dynamical. Notice also that even in the presence of a usual scalar field, the Birkhoff theorem holds up to linear perturbation around Minkowski or de Sitter background, because the scalar field does not couple to gravity in the linearized level. (It only appears quadratically

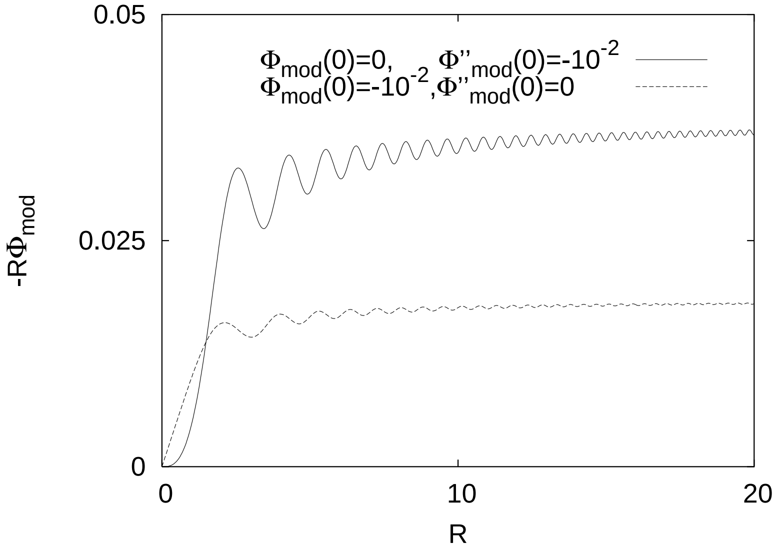

Figure 8: Numerical solutions without matter source ($\rho \equiv 0$) for the boundary condition ($\Phi_{\rm mod}(0) = 0$, $\Phi''_{\rm mod}(0) = -0.01$), and ($\Phi_{\rm mod}(0) = -0.01$, $\Phi''_{\rm mod}(0) = 0$) with $H/\Gamma = 2$.

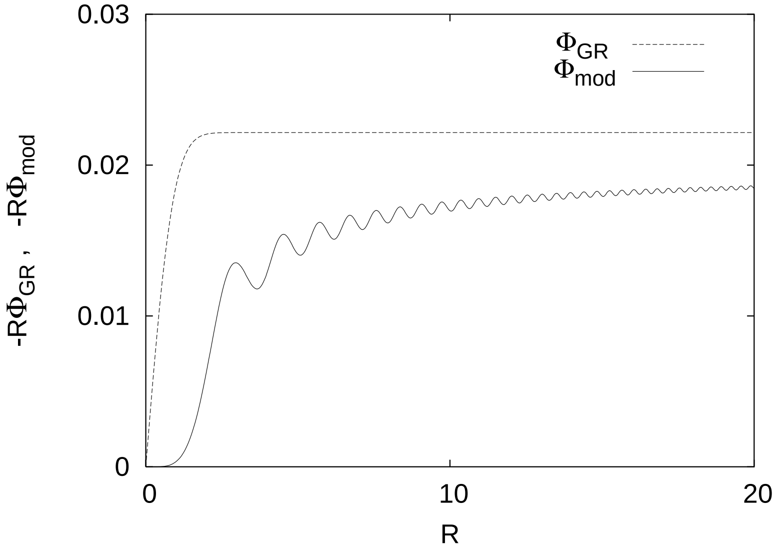

Figure 9: A numerical solution with matter source $\rho_{\rm eff}/m^2 = 0.1e^{-1.0\sigma^2}$ for the boundary condition $\Phi_{\rm mod}(0) = \Phi''_{\rm mod}(0) = 0$ and $H/\Gamma = 2$.

or higher order in $T_{\mu\nu}$.) On the other hand, in ghost condensation, π reacts to the gravitational potential produced by matter source and slowly modulates the geometry in such a way that the final stationary configuration depends on the history of the whole system. This happens even in the linear perturbation level. What figure 9 shows is just one of such possible final configurations. The reason why this remarkable modulation happens even in the linear perturbation around a de Sitter background is that we have a non-vanishing $\dot{\phi}$ in the background.

JHEP05(2004)074

9. Discussion and outlook

We have presented a consistent modification of gravity in the infrared. It is useful, though not necessary, to view this arising from gravity propagating in a ghost condensate. An immediate application of this model is an alternative to a cosmological constant for driving a de Sitter phase in the universe. This can be used to drive the acceleration of the universe today, and can also be used to for an early inflationary de Sitter phase of the universe, as will be discussed in ref. [17]. The physics here is very different from standard slow-roll inflation, and the model makes sharp predictions about the spectrum of density perturbations that can be excluded or confirmed in future experiments [17].

If the standard model fields have direct couplings to the ghost sector, there are interesting Lorentz-violating signals and inverse-square law long range forces mediated by excitations of the ghost condensate. But most interestingly, gravity is modified in the infrared in a remarkable way by the ghost condensate, giving rise to the possibility of anti-gravity and an oscillatory modulation of the newtonian potential at late time and large distances.

Our exploration of these models is still in its infancy. The most pressing question is to determine the current experimental limits on the parameters of this theory, given the unusual new forces and modifications of gravity it entails. It would also be interesting to explore further the possibility raised in section 3 that the ghost condensate may contribute to both the dark matter and the dark energy of the universe. The physics of non-linear gravitational effects, and particularly black holes in these backgrounds, is clearly interesting to explore. There are also a number of novel avenues to explore for early universe cosmology and models with extra dimensions. It would also be interesting to derive our low-energy effective theory from a more fundamental UV complete theory, either arising from ghost condensation or in some other way. Finally, can the fact that we have a model where the "energy that gravitates" is not the "particle physics energy" help with the cosmological constant problem?

Acknowledgments

We would like to thank Paolo Creminelli, Josh Erlich, Ian Low, Riccardo Rattazzi, Matthew Schwartz, and Matias Zaldarriaga for useful discussions. H.-C. C. and M. A. L. thank Aspen Center for Physics where part of the work was done. N. A.-H. and H.-C. C. are supported by NSF grant PHY-0244821, M. A. L. is supported by NSF grant PHY-009954 and S. M. is supported by NSF grant PHY-0201124. N. A.-H. is also supported in part by the David and Lucille Packard foundation.

References

[1] M. Fierz and W. Pauli, *On relativistic wave equations for particles of arbitrary spin in an electromagnetic field*, *Proc. Roy. Soc. Lond.* **A173** (1939) 211.

[2] C. Brans and R.H. Dicke, *Mach's principle and a relativistic theory of gravitation*, *Phys. Rev.* **124** (1961) 925;

R.H. Dicke, *Mach's principle and invariance under transformation of units*, *Phys. Rev.* **125** (1962) 2163;
P.G. Bergmann, *Comments on the scalar tensor theory*, *Int. J. Theor. Phys.* **1** (1968) 25.

[3] M.A. Clayton and J.W. Moffat, *Dynamical mechanism for varying light velocity as a solution to cosmological problems*, *Phys. Lett.* **B 460** (1999) 263 [astro-ph/9812481]; *Scalar-tensor gravity theory for dynamical light velocity*, *Phys. Lett.* **B 477** (2000) 269 [gr-qc/9910112]; *A scalar-tensor cosmological model with dynamical light velocity*, *Phys. Lett.* **B 506** (2001) 177 [gr-qc/0101126];
J.W. Moffat, *Bimetric gravity theory, varying speed of light and the dimming of supernovae*, *Int. J. Mod. Phys.* **D 12** (2003) 281 [gr-qc/0202012].

[4] I.T. Drummond, *Variable light-cone theory of gravity*, gr-qc/9908058; *Bimetric gravity and [dark matter]*, *Phys. Rev.* **D 63** (2001) 043503 [astro-ph/0008234].

[5] G.R. Dvali, G. Gabadadze and M. Porrati, *4d gravity on a brane in 5d Minkowski space*, *Phys. Lett.* **B 485** (2000) 208 [hep-th/0005016];
G.R. Dvali and G. Gabadadze, *Gravity on a brane in infinite-volume extra space*, *Phys. Rev.* **D 63** (2001) 065007 [hep-th/0008054];
C. Deffayet, G.R. Dvali and G. Gabadadze, *Accelerated universe from gravity leaking to extra dimensions*, *Phys. Rev.* **D 65** (2002) 044023 [astro-ph/0105068].

[6] T. Jacobson and D. Mattingly, *Gravity with a dynamical preferred frame*, *Phys. Rev.* **D 64** (2001) 024028;
D. Mattingly and T. Jacobson, *Relativistic gravity with a dynamical preferred frame*, gr-qc/0112012;
C. Eling and T. Jacobson, *Static post-newtonian equivalence of gr and gravity with a dynamical preferred frame*, *Phys. Rev.* **D 69** (2004) 064005 [gr-qc/0310044].

[7] K. Freese and M. Lewis, *Cardassian expansion: a model in which the universe is flat, matter dominated and accelerating*, *Phys. Lett.* **B 540** (2002) 1 [astro-ph/0201229].

[8] M.E. Soussa and R.P. Woodard, *A nonlocal metric formulation of MOND*, *Class. and Quant. Grav.* **20** (2003) 2737 [astro-ph/0302030]; *A generic problem with purely metric formulations of MOND*, *Phys. Lett.* **B 578** (2004) 253 [astro-ph/0307358].

[9] C. Wetterich, *Cosmology with varying scales and couplings*, hep-ph/0302116.

[10] S.M. Carroll, V. Duvvuri, M. Trodden and M.S. Turner, *Is cosmic speed-up due to new gravitational physics?*, astro-ph/0306438.

[11] J. Khoury and A. Weltman, *Chameleon fields: awaiting surprises for tests of gravity in space*, astro-ph/0309300; *Chameleon cosmology*, *Phys. Rev.* **D 69** (2004) 044026 [astro-ph/0309411].

[12] H. van Dam and M.J.G. Veltman, *Massive and massless Yang-Mills and gravitational fields*, *Nucl. Phys.* **B 22** (1970) 397;
V.I. Zakharov, *Sov. Phys. JETP Lett.* **12** (1970) 312.

[13] D.G. Boulware and S. Deser, *Inconsistency of finite range gravitation*, *Phys. Lett.* **B 40** (1972) 227.

[14] N. Arkani-Hamed, H. Georgi and M.D. Schwartz, *Effective field theory for massive gravitons and gravity in theory space*, *Ann. Phys. (NY)* **305** (2003) 96 [hep-th/0210184].

[15] M.A. Luty, M. Porrati and R. Rattazzi, *Strong interactions and stability in the dgp model*, *J. High Energy Phys.* **09** (2003) 029 [hep-th/0303116].

[16] C. Deffayet, G.R. Dvali, G. Gabadadze and A.I. Vainshtein, *Nonperturbative continuity in graviton mass versus perturbative discontinuity*, *Phys. Rev.* **D 65** (2002) 044026 [hep-th/0106001].

[17] N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, *Ghost inflation*, in preparation.

[18] R. Arnowitt, S. Deser and C. Misner, in *Gravitation: an introduction to current research*, L. Witten ed., Wiley, New York 1962.

[19] S.M. Carroll, M. Hoffman and M. Trodden, *Can the dark energy equation-of-state parameter w be less than -1?*, *Phys. Rev.* **D 68** (2003) 023509 [astro-ph/0301273].

[20] J.M. Cline, S.-y. Jeon and G.D. Moore, *The phantom menaced: constraints on low-energy effective ghosts*, hep-ph/0311312.

[21] C. Armendariz-Picon, T. Damour and V. Mukhanov, *k-inflation*, *Phys. Lett.* **B 458** (1999) 209 [hep-th/9904075];
J. Garriga and V.F. Mukhanov, *Perturbations in k-inflation*, *Phys. Lett.* **B 458** (1999) 219 [hep-th/9904176].

[22] C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *A dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration*, *Phys. Rev. Lett.* **85** (2000) 4438 [astro-ph/0004134];
C. Armendariz-Picon, V. Mukhanov and P.J. Steinhardt, *Essentials of k-essence*, *Phys. Rev.* **D 63** (2001) 103510 [astro-ph/0006373].

[23] J. Polchinski, *Effective field theory and the Fermi surface*, hep-th/9210046.

[24] D. Colladay and V.A. Kostelecky, *CPT violation and the standard model*, *Phys. Rev.* **D 55** (1997) 6760 [hep-ph/9703464]; *Lorentz-violating extension of the standard model*, *Phys. Rev.* **D 58** (1998) 116002 [hep-ph/9809521].

[25] S.R. Coleman and S.L. Glashow, *High-energy tests of Lorentz invariance*, *Phys. Rev.* **D 59** (1999) 116008 [hep-ph/9812418].

[26] V.A. Kostelecky and C.D. Lane, *Nonrelativistic quantum hamiltonian for Lorentz violation*, *J. Math. Phys.* **40** (1999) 6245 [hep-ph/9909542].

[27] A.A. Andrianov, P. Giacconi and R. Soldati, *Lorentz and CPT violations from Chern-Simons modifications of QED*, *J. High Energy Phys.* **02** (2002) 030 [hep-th/0110279].

[28] B.R. Heckel, E.G. Adelberger, J.H. Gundlach, M.G. Harris and H.E. Swanson, *Torsion balance test of spin coupled forces, Prepared for international conference on orbis scientiae 1999: quantum gravity, generalized theory of gravitation and superstring theory based unification (28th conference on high-energy physics and cosmology since 1964)*, Coral Gables, Florida, 16–19 Dec 1999.

[29] D.F. Phillips, M.A. Humphrey, E.M. Mattison, R.E. Stoner, R.F.C. Vessot and R.L. Walsworth, *Limit on Lorentz and CPT violation of the proton using a hydrogen maser*, *Phys. Rev.* **D 63** (2001) 111101.

[30] F. Canè et al., *Bound on Lorentz- and CPT-violating boost effects for the neutron*, physics/0309070.

JHEP05(2004)074

[31] R. Bluhm, *Lorentz and CPT tests in matter and antimatter*, hep-ph/0308281.

[32] A recent overview of past work on CPT and Lorentz violations can be found in, for example, *CPT and Lorentz Symmetry II*, V. A. Kostelecky, ed., World Scientific, Singapore 2002.

[33] This definition of quasi-local mass in spherical symmetric spacetime is known as Misner-Sharp mass: C.W. Misner and D.H. Sharp, *Relativistic equations for adiabatic, spherically symmetric gravitational collapse*, *Phys. Rev.* **136** (1964) B571.

[34] G.D. Birkhoff and R.E. Langer, *Relativity and modern physics*, Harvard University Press, Cambridge 1923.

JHEP05(2004)074